Mathematics V1207x
Honors Mathematics A

Midterm Examination

October 28, 2015

READ AND FOLLOW CAREFULLY ALL INSTRUCTIONS BELOW

Turn off all electronic devices.

Brief visits to the men's or women's room (just out the door to your left) are OK, but one at a time only.

Write your name, "Honors Math A, Prof. Thaddeus," and the number of blue books on the cover of each blue book.

Write your name on the attendance sheet when it comes around.

Write all answers and work in your blue books. Do not hand in this sheet.

On each page you use, write the number of the problem *in a circle in the margin.*

You may do more than one problem on a single page. Just put all numbers by their problems.

You may do the problems out of order, but this is discouraged as it can lead to misgrading.

When there is any doubt, state briefly but clearly what statements from Apostol, lecture, or assignments you are using.

You may use, without comment, facts from logic and high-school algebra, and basic facts about the natural and real numbers (such as those stated without proof in Apostol).

In grading the exams, I will emphasize accuracy, brevity, and clarity. Aim for all three.

Attempt all 7 problems. Each is worth 10 points. Good luck!

1. Define (a) the *lower* and *upper integral*; (b) an *integrable* function.

2. Let $f, g : \mathbb{R} \to \mathbb{R}$ be any two functions with $\lim_{x\to c} f(x) = \lim_{x\to c} g(x) = k$.

Let $h(x) = \begin{cases} f(x) & \text{if } x \in \mathbb{Q} \\ g(x) & \text{if } x \notin \mathbb{Q}. \end{cases}$

Prove that $\lim_{x\to c} h(x) = k$.

3. Prove that $f : \mathbb{R} \to \mathbb{R}$ is continuous if and only if $g(x) = f(1-x)$ is continuous.

4. Prove that if $f : [a, b] \to \mathbb{R}$ is integrable and $f \geq 0$, then its indefinite integral is increasing.

5. Prove that if c is an upper bound for $S \subset \mathbb{R}$ and $c \in S$, then $c = \sup S$.

6. Define (a) a *finite* set; (b) an *infinite* set.

7. Prove that a nonempty finite subset of $\mathbb{R}$ contains its supremum.