Mathematics V1207x
Honors Mathematics A

Assignment #7
Due November 6, 2015

Reading: Apostol, §§4.10–4.20, pp. 174–194, and §§5.1–5.6, pp. 202–211.

1. If $f : [a, b] \to \mathbb{R}$ is continuous and $f(x) = 0$ whenever x is rational, prove that $f(x) = 0$ for all $x \in [a, b]$.

2. If $f : [a, b] \to \mathbb{R}$ and $g : [a, b] \to \mathbb{R}$ are both continuous and $f(x) = g(x)$ whenever x is rational, prove that $f = g$.

***3.** Let $f : [a, b] \to \mathbb{R}$ be any integrable function. Prove that there is some $c \in [a, b]$ such that

$$\int_a^c f(x)\,dx = \tfrac{1}{2}\int_a^b f(x)\,dx.$$

(Hint: consider the properties of the indefinite integral as a function $[a, b] \to \mathbb{R}$.)

***4.** The intermediate value theorem was used in class to show that for every integer $n > 0$, every real $x \geq 0$ has an unique nonnegative nth root, that is, a real $y \geq 0$ such that $y^n = x$. Denote y by $\sqrt[n]{x}$ or $x^{1/n}$. Generalize the argument given in class for $n = 2$ to show that the function $f : [0, \infty) \to \mathbb{R}$ given by $f(x) = \sqrt[n]{x}$ is continuous.

5. Apostol §3.11 (p. 145) 1, 3, *5, 6.

6. Apostol §3.20 (p. 155) *7, *8.

7. Apostol §4.6 (p. 168) *24.

8. Apostol §4.9 (p. 173) 9, *15. (In 15, each reason should be a very brief sketch of a proof or a counterexample. Also, there is a misprint: the denominators in (c) and (d) should be $2t$ and t respectively, not the other way around.)

9. Apostol §4.12 (p. 180) 16, *19bc.