Mathematics V1207x
Honors Mathematics A

Assignment #6
Due October 23, 2015

Reading: Apostol, §§3.9–3.19, pp. 142–154, and §§4.1–4.8, pp. 156–172.

1. Apostol §3.6 (pp. 138–140) *15, 20, 28, *31, *33. (In 31, if you give an example, prove that it works. For 33, don't neglect part c on p. 140.)

2. Apostol §3.8 (p. 142) *11, 12, *16, *17, 18, 19. (Don't use l'Hôpital's rule.)

***3.** Let $f : (a, b) \to \mathbb{R}$ and assume $x \in (a, b)$.

Consider the following two statements:

(a) $\lim_{h\to 0} |f(x + h) - f(x)| = 0$;

(b) $\lim_{h\to 0} |f(x + h) - f(x - h)| = 0$.

Show that (a) implies (b). Also give a counterexample to show that the converse statement, (b) implies (a), need not be true.

***4.** (a) Show that a monotonic surjective function $f : \mathbb{R} \to \mathbb{R}$ must be continuous.

(b) Give an example of a function $f : \mathbb{R} \to \mathbb{R}$ which is monotonic but not continuous. Prove both of these statements.

5. Let f be defined on $[0, 1]$ as follows:

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is irrational} \\ 1/n & \text{if } x \text{ is the rational number } m/n \text{ in lowest terms.} \end{cases}$$

(a) Show that f is continuous at x if and only if x is irrational.

(b) Show that $\int_0^1 f(x)\,dx = 0$.