Mathematics V1207x
Honors Mathematics A

Assignment #4
Due October 9, 2015

Reading: Apostol, 1.12–1.27, pp. 64–85, and 2.1–2.7, pp. 88–104.

1. Apostol §1.11 (pp. 63–64) 2, 4a, *4b, 4d, 5. (Here the greatest integer $[x]$ is defined to be the unique integer n such that $n \le x < n+1$. You supposedly proved that this exists in an unstarred problem on Assignment #3.)

2. Apostol §1.15 (pp. 70–72) 1, 3, *5, 10, *11, *15, 16, 17. (In 5, you may assume that every nonnegative number has a unique nonnegative square root, as proved in Theorem I.35. In 11, if true, give the briefest possible sketch of the proof; if false, give a counterexample.)

***3.** Prove the reflection property of integration: if f is integrable on $[a, b]$, then

$$\int_{-b}^{-a} f(-x)\,dx = \int_a^b f(x)\,dx.$$

(First prove it for step functions.)

4. Apostol §1.26 (pp. 83–84) *25, 27.

***5.** Prove that if f and $|f|$ are both integrable on $[a, b]$, then

$$\left|\int_a^b f(x)\,dx\right| \le \int_a^b |f(x)|\,dx.$$

Notice the similarity to the triangle inequality!
(Hint: you may find it convenient to use **8** from Assignment #3.)

6. Is it possible for $|f|$ to be integrable but not f? Give a proof or counterexample.