Mathematics V1207x
Honors Mathematics A

Assignment #2
Due September 25, 2015

Read the rest of the Introduction in Apostol (skipping starred sections if you wish).

Then do the following problems. As before, hand in only the starred ones.

If the later exercises (from **5** onward) do not make sense, then either read ahead in Apostol or wait until Wednesday's lecture.

***1.** (a) Prove that if A and B are sets, then their power sets satisfy

$$\mathcal{P}(A) \cup \mathcal{P}(B) \subset \mathcal{P}(A \cup B).$$

(b) Give an example, however, to show that equality need not hold in the above.

***2.** Use the definition of ordered pair equality to prove that if A, B, C are sets, then

$$A \times (B \cup C) = (A \times B) \cup (A \times C).$$

3. If $f : S \to T$ and $g : T \to U$ are injective functions, prove that the composite $g \circ f$ is also injective.

***4.** Suppose that $f : S \to T$ and $g : T \to S$ are functions such that $g \circ f = \mathrm{id} : S \to S$. (In this case, we say that g is a *left inverse* for f.) For each of the following, give a proof if true, or a counterexample if false. (a) f is injective; (b) f is surjective; (c) g is injective; (d) g is surjective.

***5.** If $S \subset \mathbf{R}$ and $T \subset \mathbf{R}$ are inductive sets, prove that $S \cap T$ is too.

6. Prove that every positive integer is positive! That is, if n is a positive integer, then $n > 0$.

***7.** Prove that the product of two positive integers is a positive integer. Hint: induction. Prove that the product of two integers is an integer.

Also do the following problems from Apostol.

Apostol §I 3.3 (p. 19) 1 (do I.6–8), 3, 4*.

Apostol §I 3.5 (p. 21) 1 (do I.24*), 2, 5, 9, 10*.

Apostol §I 4.4 (pp. 35–36) 1ab, 1c*, 4, 12.