Mathematics V1208y
Honors Mathematics IV
Assignment #10
Due April 14, 2003

Reading: Apostol, §§11.1–11.17 (pp. 353–377).


2. Consider the vector field on the open first quadrant defined by

\[ F(x, y) = \left( \frac{y + 1}{x^2 y}, \frac{x + 1}{xy^2} \right). \]

Is it conservative? Why or why not? If so, what are all the possible potentials?

*3. Prove the fact, asserted in lecture, that given a closed rectangle \( Q \subset \mathbb{R}^n \), a partition of \( Q \), and a point \( x \in Q \), there exists \( \delta > 0 \) such that every \( z \in Q \) with \( \|z - x\| < \delta \) lies in some closed subrectangle of the partition which also contains \( x \).

*4. (a) Give an example of a bounded function \( f : [0, 1] \times [0, 1] \to \mathbb{R} \) which is not integrable. (Of course, you should prove that it’s not integrable...)

(b) Give an example of an integrable function \( g : [0, 1] \times [0, 1] \to \mathbb{R} \) such that for all fixed \( x \), \( \int_0^1 g(x, y) \, dy \) exists, but for some fixed \( y \), \( \int_0^1 g(x, y) \, dx \) does not exist. Hint: it could even be a step function.

(c) Give an example of a bounded function \( h : [0, 1] \times [0, 1] \to \mathbb{R} \) such that for all fixed \( x \), \( \int_0^1 h(x, y) \, dy \) exists, but the iterated integral \( \int_0^1 \int_0^1 h(x, y) \, dy \, dx \) does not exist. Challenge problem: can such a function be integrable?