Put all answers and work in your blue books. On each page you use, put the number of the problem inside a circle in the margin. You may do the problems out of order, but this is discouraged. Always state briefly but clearly what statements from Apostol or lecture you are using. You may use without comment facts from logic and high-school algebra, such as $1 + 1 = 2$. All problems are worth 10 points. Good luck!

1. State the axiom of powers.

2. Prove that if $f : S \to T$ and $g : T \to U$ are surjective, then so is $g \circ f : S \to U$.

3. Prove that for all $n \in \mathbb{N}$, $\sum_{i=1}^{n} 2^{i-1} = 2^n - 1$.

4. Prove that if an integrable $f : [a, b] \to \mathbb{R}$ satisfies $f \geq 0$, then the indefinite integral $g : [a, b] \to \mathbb{R}$ (given as usual by $g(x) = \int_{a}^{x} f(t) \, dt$) is increasing.

5. For any integrable $f : [a, b] \to \mathbb{R}$, show there exists a step function $s : [a, b] \to \mathbb{R}$ such that $s \leq f$ and

$$\int_{a}^{b} (f - s)(x) \, dx < 1.$$ 

6. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 2x$ if $x$ is rational, $f(x) = x$ if $x$ is irrational. Show that $f$ is continuous at 0.

7. Let $f : [1, 4] \to \mathbb{R}$ be $f(x) = x^3 - 5\sqrt{x}$. Show that $f$ is bounded and that there exists $c \in (1, 4)$ such that $f(c) = 0$. 