Mathematics V1207x  
Honors Mathematics III  
Assignment #3  
Due September 27, 2002

Reading: Apostol 1.1–1.10, pp. 48–63.

1. Apostol §I 3.12 (pp. 28–29) #1, 3*, 4, 6*, 7, 10, 11. (Comment: #4 is surprisingly challenging; don’t be discouraged if you can’t do it. It gets much easier if you assume the well-ordering principle from I 4.3.)

2. Apostol §I 4.9 (p. 43) #1bdrg*j*.

3. Suppose $S \subseteq \mathbb{R}$ and $t \in \mathbb{R}$. Show that $t = \sup S$ if and only if both of the following are true: (a) $t$ is an upper bound for $S$, and (b) for all $\varepsilon > 0$, there exists $x \in S$ such that $x > t - \varepsilon$.

*4. Suppose that $S, T \subseteq \mathbb{R}$, both $S$ and $T$ are nonempty and bounded above, and there is a bijective function $f : S \to T$ such that $x \geq f(x)$ for all $x \in S$. Show that $\sup S \geq \sup T$. What can you say if you only know $x > f(x)$ for all $x \in S$?

*5. Prove that the Cartesian product of two finite sets is finite. (Hint: Consider first the case $S = \{1, \ldots, m\}$ and $T = \{1, \ldots, n\}$ and try induction on $m$. Then deduce the general case from this one.)

6. Prove that the intersection and union of two finite sets is finite. (You’ll have to define a function to a subset of $\mathbb{N}$ and show that it is bijective. It might be convenient to use the well-ordering principle again.)

As always, only starred problems are to be handed in. But you may use unstarred problems in the solutions of starred problems.