Mathematics V1208y
Honors Mathematics B

Midterm Examination

March 9, 2016

READ AND FOLLOW CAREFULLY ALL INSTRUCTIONS BELOW

Turn off all electronic devices.
Brief visits to the men's or women's room are OK, but one at a time only, and please seek permission first.
Write your name, "Honors Math B, Prof. Thaddeus," and the number of blue books on the cover of each blue book.
Write your name on the attendance sheet when it comes around.
Write all answers and work in your blue books. Do not hand in this sheet.
On each page you use, write the number of the problem *in a circle in the margin.*
You may do more than one problem on a single page. Just put all numbers by their problems.
You may do the problems out of order, but this is discouraged as it can lead to misgrading.
When there is any doubt, state briefly but clearly what statements from Apostol, lecture, or assignments you are using.
In grading the exams, I will emphasize accuracy, brevity, and clarity. Aim for all three.
Attempt all 7 problems. Each is worth 10 points. Good luck!

1. State the definition of reduced row-echelon form.

2. (a) Diagonalize $A = \begin{pmatrix} -4 & 6 \\ -3 & 5 \end{pmatrix}$.

(b) Give an explicit formula, depending on k, for the upper right-hand entry of A^k.

3. (a) Find the determinant of $B = \begin{pmatrix} 4 & 1 & 0 & 0 & 0 & 0 \\ 5 & 0 & 1 & 0 & 0 & 0 \\ 6 & 0 & 0 & 1 & 0 & 0 \\ 7 & 0 & 0 & 0 & 1 & 0 \\ 8 & 0 & 0 & 0 & 0 & 1 \\ 9 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$. Explain your reasoning.

(b) Find the rank of B^7. Explain your reasoning.

4. Let C be the $n \times n$ matrix such that $C_{ij} = i + j$.
Use row operations to show that $\det C = 0$ if and only if $n > 2$.

5. Let S and T be linear maps $\mathbb{R}^n \to \mathbb{R}^n$ such that $S \circ T = 0$.

(a) Prove that $\operatorname{im} T \subset \ker S$.

(b) Prove that $\operatorname{rank} S + \operatorname{rank} T \leq n$.

6. Let D be an $n \times n$ matrix such that $D^2 = D$.

(a) Prove that the eigenvalues of D are all either 0 or 1.

(b) Prove that the image of T_D equals its 1-eigenspace.

(c) Optional challenge problem (not for credit): Prove that D is diagonalizable.

7. If $E \in M_{n\times n}$ has n distinct eigenvalues $\lambda_1, \ldots, \lambda_n$, prove that $\det E = \lambda_1 \lambda_2 \cdots \lambda_n$.