Mathematics V1208y
Honors Mathematics B

Assignment #9
Due April 8, 2016

Reading: Apostol, chapter 10.

Remark on notation: In the plane, Apostol often writes $\int_C f\,dx + g\,dy$ for the line integral that was expressed in lecture as $\int_C (f,g)\cdot d\gamma$. It makes sense, as $\frac{d\gamma}{dt} = \left(\frac{dx}{dt}, \frac{dy}{dt}\right)$, so at least formally, $d\gamma = (dx, dy)$.

1. Apostol §8.17 (pp. 268–9) *12.

Hint: compose with a suitable curve and use the chain rule.

2. Apostol §8.22 (pp. 275–7) *3ab (and evaluate explicitly for $X(s,t) = s+t$, $Y(s,t) = st$, and $f(x,y) = e^{x-y}$), 8, 9, *14, 15.

3. Apostol §8.24 (pp. 281–2) *4.

4. Apostol §10.5 (p. 328) 2, 7, 8, *10.

***5.** Let $U = \{(x,y) \in \mathbb{R}^2 \,|\, (x,y) \neq (0,0)\}$ and let $F : U \to \mathbb{R}^2$ be given by

$$F(x,y) = \left(\frac{x+y}{x^2+y^2}\,,\, \frac{y-x}{x^2+y^2}\right).$$

Use the previous problem to show that this is not conservative.

6. Apostol §10.9 (pp. 331–2) 1, 8, *10.

7. Apostol §10.13 (pp. 336–7) 4, *5a, 6, 7.

8. Apostol §10.18 (pp. 345–6) *14.

***9.** A radial force field in $\mathbb{R}^n$ may be expressed as $F(\mathbf{r}) = f(\|\mathbf{r}\|)\mathbf{r}/\|\mathbf{r}\|$. Assuming that f is a smooth function of one variable, show that F is conservative.