Mathematics V1208y
Honors Mathematics B

Assignment #8
Due April 1, 2016

1. Apostol §8.3 (pp. 245–7) 2. You needn't do it all, just enough to get the idea.

2. Apostol §8.5 (pp. 251–2) *6, 7, 8.

3. Apostol §8.9 (pp. 255–6) 4, 7, 8, 9, 10, 12, *20, 21, *22.

4. Apostol §8.14 (pp. 262–3) 1, 2, 7, 10, 11.

5. Apostol §8.24 (pp. 281–2) *1, 2, 3, *12a, *13.

Hint for 13: you may use the fact, asserted in class and in Figure 8.8, that for any scalar field $f : \mathbb{R}^3 \to \mathbb{R}$, its gradient vectors $\nabla f(x, y, z)$ are perpendicular to the tangent planes of its level surfaces $f(x, y, z) = c$. We will speak more rigorously about surfaces and tangent planes in the future.

6. Show that if U and V are open in $\mathbb{R}^n$, then so are $U \cup V$ and $U \cap V$.

***7.** Show that the product $(a_1, b_1) \times (a_2, b_2) \times \cdots \times (a_n, b_n)$ is an open set in $\mathbb{R}^n$.

***8.** Suppose that $F : \mathbb{R}^n \to \mathbb{R}$ is linear. Show that for any $\mathbf{x} \in \mathbb{R}^n$, the total derivative of F at $\mathbf{x}$ is just F itself.

***9.** A function $G : \mathbb{R}^n \to \mathbb{R}$ is called *homogeneous*[*] if $G(t\mathbf{x}) = t\,G(\mathbf{x})$ for all nonzero $t \in \mathbb{R}$ and all nonzero $\mathbf{x} \in \mathbb{R}^n$. Suppose that G is homogeneous and continuous.

(a) Show that $G(\mathbf{0}) = 0$.

(b) Show that the directional derivative of G at $\mathbf{0}$ along $\mathbf{y}$ exists for all $\mathbf{y} \in \mathbb{R}^n$.

(c) Show that G is differentiable at $\mathbf{0}$ if and only if it is linear. Hint: If it's differentiable, look at a single line through the origin at a time, and show that there it equals its own derivative.

***10.** Suppose $H : \mathbb{R}^n \to \mathbb{R}$ satisfies $|H(\mathbf{x})| \leq c\,\|\mathbf{x}\|^2$ for some constant $c \in \mathbb{R}$ and for all $\mathbf{x} \in \mathbb{R}^n$. Show that it is differentiable at $\mathbf{0}$ and compute its derivative.

[*] Strictly speaking, this is really called *homogeneous of degree 1*, but never mind.