Mathematics V1208y
Honors Mathematics B

Assignment #5
Due February 26, 2016

Reading: Apostol §§5.1–5.10 (pp. 114–124) and §5.19 (pp. 138–141).

***1.** For each of the matrices below, either diagonalize it (i.e. express it as BDB^{-1} with D diagonal) or prove that this is impossible. Hint: it helps to choose cleverly when expanding by minors.

$$\text{(a)} \begin{pmatrix} 20 & -9 \\ 30 & -13 \end{pmatrix}; \text{ (b)} \begin{pmatrix} 8 & 4 \\ -9 & -4 \end{pmatrix}; \text{ (c)} \begin{pmatrix} -1 & 4 & 4 \\ 0 & -5 & -4 \\ 0 & 8 & 7 \end{pmatrix}.$$

2. Apostol §4.4 (p. 101) 1, 2, 3, *4, 6, 10, 11, 12.

3. Apostol §4.8 (pp. 107–8) 1, 2, 7, *11.

4. Apostol §4.10 (pp. 112–13) 2, 4, 6, *7, *8abc, 8d.

***5.** Let V be an n-dimensional vector space and let $T : V \to V$ be a linear map. Define the *characteristic polynomial* $\chi_T : \mathbb{R} \to \mathbb{R}$ by $\chi_T(\lambda) = \det(\lambda I - A)$, where A is the matrix representing T in any basis.

(a) Prove that χ_T does not depend on the choice of basis.

(b) Prove that χ_T is a polynomial function of λ of degree n with leading term λ^n.

***6.** If A is an upper-triangular square matrix, show that the eigenvalues of A are exactly its diagonal entries a_{ii}. (“Exactly” means a number is an eigenvalue if and only if it's one of the diagonal entries.)

7. Let P_k be the space of polynomials of degree $\leq k$ as usual, and consider the maps $T : P_k \to P_k$ given by $T(p) = q$, where

(a) $q(x) = p'(x)$;

(b) $q(x) = x\,p'(x)$;

(c) $q(x) = p''(x) + p'(x) + p(x)$;

(d) $q(x) = p(x + 1)$.

In each case, find the eigenvalues, eigenfunctions (that is, functions that are eigenvectors) and determinant of T. In which cases is it diagonalizable?