Mathematics V1208y
Honors Mathematics B

Assignment #3

Due February 12, 2016

Buy Apostol Vol. II if you haven't already and read §3.

1. Apostol Vol. I §16.16 (pp. 603–4) = Vol. II §2.16 (pp. 57–58) 6, 7, 8, 11.

2. Apostol Vol. I §16.20 (pp. 613–14) = Vol. II §2.20 (pp. 67–68) 7, 8, 9, *11, *12, 15.

For 12, use the method outlined in lecture and in Apostol. If you happen to know Cramer's rule, don't use it.

3. Apostol Vol. I §16.21 (pp. 614–15) = Vol. II §2.21 (pp. 68–70) *1, *2abcf, 2de, 5, 7ac, *7bde.

Singular here means noninvertible, so *nonsingular* means invertible. For 1 and 2, use the facts proved in lecture and in §2 of Apostol; don't use determinants.

Hint for 7: if A has transpose A^t, then $A^t_{ij} = A_{ji}$.

***4.** For matrices $A \in M_{m\times n}$, $B \in M_{m\times n}$, $C \in M_{n\times p}$, prove that $(A+B)C = AC + BC$.

***5.** Use Gauss-Jordan elimination and back substitution to find the general solution (i.e. the set of all real solutions) to the following linear systems. Show your work.

$$\text{(a)} \begin{cases} 2x_1 + 4x_2 + 8x_3 + 6x_4 &= 0 \\ 5x_1 + 6x_2 + 8x_3 + 7x_4 &= 0 \\ 6x_1 + 7x_2 + 9x_3 + 8x_4 &= 0 \\ 5x_1 + 4x_2 + 2x_3 + 3x_4 &= 0 \end{cases} \qquad \text{(b)} \begin{cases} 2x_1 + 4x_2 + 8x_3 + 6x_4 &= 2 \\ 5x_1 + 6x_2 + 8x_3 + 7x_4 &= 9 \\ 6x_1 + 7x_2 + 9x_3 + 8x_4 &= 11 \\ 5x_1 + 4x_2 + 2x_3 + 3x_4 &= 11 \end{cases}$$

$$\text{(c)} \begin{cases} 2x_1 + 4x_2 + 8x_3 + 6x_4 &= 2 \\ 5x_1 + 6x_2 + 8x_3 + 7x_4 &= 9 \\ 6x_1 + 7x_2 + 9x_3 + 8x_4 &= 6 \\ 5x_1 + 4x_2 + 2x_3 + 3x_4 &= 11 \end{cases}$$

***6.** A square matrix is said to be *symmetric* if it equals its transpose, $A = A^t$, and *skew-symmetric* if $A = -A^t$. For example, the identity matrix is symmetric, since $\delta_{ij} = \delta_{ji}$.

(a) Prove that if A is symmetric and invertible, then its inverse is symmetric.
Give a 2×2 example.

(b) Prove that if A is skew-symmetric and invertible, then its inverse is skew-symmetric.
Give a 2×2 example.

7. A square matrix A is said to be *upper-triangular* if $A_{ij} = 0$ whenever $i > j$. Prove that if A is upper-triangular and invertible, then its inverse is upper-triangular. Give a 2×2 example.

***8.** If A and B are $n \times n$ matrices, prove that $\operatorname{rank} AB \le \min(\operatorname{rank} A, \operatorname{rank} B)$.