Mathematics V1208y
Honors Mathematics B
Assignment #11
Due April 22, 2016

Reading: Apostol, §11.26–11.33 (pp. 392–413) and §§12.1–12.9 (pp. 417–436).


2. (a) Prove the comparison theorem for multiple integrals: if $Q$ is a closed rectangle, $f$ and $g$ are integrable on $Q$, and $f \leq g$, then $\iint_Q f \leq \iint_Q g$.
   
   (b) Same as (a), except that the domain of integration is an arbitrary bounded $S \subset \mathbb{R}^n$.
   
   (c) If $S \subset T \subset \mathbb{R}^n$ are bounded, and if $f \geq 0$ is integrable on both $S$ and $T$, show that $\iint_S f \leq \iint_T f$.

*3. Suppose $f : Q \to \mathbb{R}$ is continuous, where $Q \subset \mathbb{R}^{n-1}$ is a closed rectangle, and (for some fixed index $i$)

$$P = \{(x_1, \ldots, x_n) \in \mathbb{R}^n | (x_1, \ldots, \hat{x}_i, \ldots) \in Q \text{ and } x_i = f(x_1, \ldots, \hat{x}_i, \ldots)\}.$$

Show that for any continuous function $\phi : \mathbb{R}^n \to \mathbb{R}$, $\iint_P \phi = 0$. Sketch in $\mathbb{R}^2$ or $\mathbb{R}^3$.

*4. (a) Show that if $S$ and $T$ are disjoint subsets of $\mathbb{R}^n$, and if $\phi : \mathbb{R}^n \to \mathbb{R}$ is a continuous function such that $\iint_S \phi$ and $\iint_T \phi$ exist, then $\iint_{S\cup T} \phi$ exists and equals the sum of the previous two integrals. Hint: use linearity for multiple integrals.
   
   (b) Same as (a), except that $S$ and $T$ now intersect in a set $P$ as in 1 above. You may regard this as a concatenation theorem for multiple integrals.

5. Let $a, b, c, d$ be continuous functions $[0, 1] \to \mathbb{R}$ with $a < b$ and $c < d$, and let $Q(t)$ be the rectangle $[a(t), b(t)] \times [c(t), d(t)]$. Show that if $f : \mathbb{R}^2 \to \mathbb{R}$ is a continuous function, then $\iint_{Q(t)} f$ is a continuous function of $t$.

6. Apostol §11.22 (pp. 385–7) *1ad, 2, 4, *5, 6, 7, 8ab. (In 5, the curve $C$ does not really have to be a simple closed curve; it could be any piecewise $C^1$ closed curve.)

7. Apostol §11.28 (pp. 399–401) 8, *9, *14, *16abc. This last computation is a classic: “A mathematician is one to whom that is as obvious as that twice two makes four is to you” —Lord Kelvin. We didn’t define $\int_0^\infty$ last semester but it just means $\lim_{r \to \infty} \int_0^r$.

8. Apostol §11.34 (pp. 413–6) 1, 7, 18, 33a.