Mathematics V1208y
Honors Mathematics B

Assignment #11
Due April 22, 2016

Reading: Apostol, §11.26–11.33 (pp. 392–413) and §§12.1–12.9 (pp. 417–436).

1. Apostol §11.15 (pp. 371–3) *7, 11, *14, 15.

2. (a) Prove the comparison theorem for multiple integrals: if Q is a closed rectangle, f and g are integrable on Q, and $f \le g$, then $\iint_Q f \le \iint_Q g$.

(b) Same as (a), except that the domain of integration is an arbitrary bounded $S \subset \mathbb{R}^n$.

(c) If $S \subset T \subset \mathbb{R}^n$ are bounded, and if $f \ge 0$ is integrable on both S and T, show that $\iint_S f \le \iint_T f$.

***3.** Suppose $f : Q \to \mathbb{R}$ is continuous, where $Q \subset \mathbb{R}^{n-1}$ is a closed rectangle, and (for some fixed index i)

$$P = \{(x_1, \ldots, x_n) \in \mathbb{R}^n \,|\, (x_1, \ldots, \hat{x}_i, \ldots) \in Q \text{ and } x_i = f(x_1, \ldots, \hat{x}_i, \ldots)\}.$$

Show that for any continuous function $\phi : \mathbb{R}^n \to \mathbb{R}$, $\iint_P \phi = 0$. Sketch in $\mathbb{R}^2$ or $\mathbb{R}^3$.

***4.** (a) Show that if S and T are disjoint subsets of $\mathbb{R}^n$, and if $\phi : \mathbb{R}^n \to \mathbb{R}$ is a continuous function such that $\iint_S \phi$ and $\iint_T \phi$ exist, then $\iint_{S \cup T} \phi$ exists and equals the sum of the previous two integrals. Hint: use linearity for multiple integrals.

(b) Same as (a), except that S and T now intersect in a set P as in **1** above. You may regard this as a concatenation theorem for multiple integrals.

5. Let a, b, c, d be continuous functions $[0, 1] \to \mathbb{R}$ with $a < b$ and $c < d$, and let $Q(t)$ be the rectangle $[a(t), b(t)] \times [c(t), d(t)]$. Show that if $f : \mathbb{R}^2 \to \mathbb{R}$ is a continuous function, then $\iint_{Q(t)} f$ is a continuous function of t.

6. Apostol §11.22 (pp. 385–7) *1ad, 2, 4, *5, 6, 7, 8ab. (In 5, the curve C does not really have to be a simple closed curve; it could be any piecewise C^1 closed curve.)

7. Apostol §11.28 (pp. 399–401) 8, *9, *14, *16abc. This last computation is a classic: "A mathematician is one to whom *that* is as obvious as that twice two makes four is to you" —Lord Kelvin. We didn't define $\int_0^\infty$ last semester but it just means $\lim_{r\to\infty} \int_0^r$.

8. Apostol §11.34 (pp. 413–6) 1, 7, 18, 33a.