Mathematics V1208y
Honors Mathematics B

Assignment #10
Due April 15, 2016

Reading: Apostol, §11.1–11.21 (pp. 353–385).

Except for concatenation, all the basic properties of the one-variable Riemann integral (such as linearity) carry over straightforwardly to multiple integrals, both in their statements and in their proofs. You may assume them. An analogue of concatenation may appear on the next assignment.

1. Apostol §10.18 (pp. 345–6) *3, *9, 11, *17, *18.

Note that 17 and 18 refer to the vector field defined at the bottom of p. 345. You may solve 18 following Apostol if you wish, but you may also skip steps (a) and (b) and prove more easily that f is a gradient on T by quoting a theorem from class.

2. Consider the vector field on the open first quadrant defined by

$$F(x, y) = \left(\frac{y+1}{x^2 y}, \frac{x+1}{xy^2} \right).$$

Is it conservative? Why or why not? If so, what are all the possible potentials?

***3.** (a) Give an example of a bounded function $f : [0,1] \times [0,1] \to \mathbb{R}$ which is *not* integrable. (Of course, you should *prove* that it's not integrable. . .)

(b) Give an example of an integrable function $g : [0,1] \times [0,1] \to \mathbb{R}$ such that for all fixed x, $\int_0^1 g(x,y)\, dy$ exists, but for some fixed y, $\int_0^1 g(x,y)\, dx$ does *not* exist. Hint: it could even be a step function.

(c) Give an example of a bounded function $h : [0,1] \times [0,1] \to \mathbb{R}$ such that for all fixed x, $\int_0^1 h(x,y)\, dy$ exists, but the iterated integral $\int_0^1 \int_0^1 h(x,y)\, dy\, dx$ does *not* exist. Challenge problem: can such a function be integrable?

4. Apostol §11.9 (pp. 362–3) *1, 5, *9, *14.

5. Apostol §11.15 (pp. 371–3) *1, 4, 5.

Note: in 5, "$[0, \pi]$" really means $[0, \pi] \times \{0\}$.

***6.** Give a counterexample to show that uniform continuity is false for a continuous function on an open rectangle.