1. Let $M$ be $\mathbb{R}^4$, regarded as the set of $2 \times 2$ matrices with real entries, and define an action of $\mathbb{R}$ on $M$ by
\[ t \cdot A = \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix} \cdot A. \]
Find the infinitesimal generator.

2. Let $M = \mathbb{R}^n$ and consider vector fields $F : M \to \mathbb{R}^n$, $G : M \to \mathbb{R}^n$. Give an explicit formula for the Lie bracket $[F,G] : M \to \mathbb{R}^n$.

3. Show that there are exactly two 2-dimensional Lie algebras up to isomorphism. Describe each of them as a subalgebra of $\mathfrak{gl}(2, \mathbb{R})$.

4. Suppose $M$ is a manifold embedded in $\mathbb{R}^N$. Show that there is a natural, bijective correspondence between vector fields $VF(M)$, as defined in class, and smooth maps $F : M \to \mathbb{R}^N$ such that for each $p \in M$, $F(p) \in T_pM \subset \mathbb{R}^N$.

5. Let $\gamma : (-a,b) \to M$ be an integral curve for the flow of $X \in VF(M)$. Recall that $\gamma'(t) := D_t \gamma(1)$.
   (a) Show that either $\gamma'(t) = 0$ for all $t \in (-a,b)$ or $\gamma'(t) \neq 0$ for all $t \in (-a,b)$.
   (b) Show that every nonconstant integral curve is an immersion.

6. If $H$ is a Lie group and $F : G \to H$ a connected covering space (so that $G$ is also a Lie group), show that the induced map $f : \mathfrak{g} \to \mathfrak{h}$ is an isomorphism of Lie algebras.

7. Let $G$ be a Lie group.
   (a) Compute the derivative at $(e,e)$ of the multiplication $m : G \times G \to G$.
   (b) Compute the derivative at $e$ of the inversion $i : G \to G$.
   (c) Show that $G$ is abelian if and only if $i$ is a homomorphism.
   (d) Show that if $G$ is abelian, then its Lie algebra $\mathfrak{g}$ has zero bracket, $[X,X] = 0$.
   (e) Give a counterexample to the converse statement.

8. The support of a vector field $X \in VF(M)$ is the closure of $\{p \in M \mid V(p) \neq 0\}$. If $X$ has compact support, show that its flow is complete (that is, defined on all of $\mathbb{R} \times M$).

9. Show that the group of diffeomorphisms acts transitively on any connected manifold: that is, for any $p,q \in M$ connected, there exists a diffeomorphism $f : M \to M$ such that $f(p) = q$. Hint: first show that, if $p,q$ lie in an embedded open ball, there is a compactly supported flow taking $p$ to $q$. 