Mathematics G4402x
Modern Geometry
Fall 2013

Instructor:	Prof. Michael Thaddeus	**Lectures:**	M.W. 4:10–5:25
Office:	Mathematics 414	**Office hours:**	M.W. 5:30–6
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Prerequisites: Math W4051x, *Topology* (fundamental group, covering spaces) and Math W4081y, *Differentiable Manifolds* (implicit function theorem, manifolds in $\mathbf{R}^n$, differential forms, generalized Stokes's theorem). A less advanced differential geometry course is Math V3386y, *Differential Geometry*.

Texts: William M. Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry*; Raoul Bott & Loring Tu, *Differential Forms in Algebraic Topology*; John M. Lee, *Introduction to Smooth Manifolds*; Frank W. Warner, *Foundations of Differentiable Manifolds and Lie Groups*. All have been placed on reserve in the Mathematics Library.

Course outline: The elements of differential topology — smooth manifolds, tangent spaces, immersions and embeddings, Lie groups and their actions, homogeneous spaces, vector fields, the fundamental theorem on flows, Frobenius integrability theorem, differential forms and Stokes's theorem, de Rham cohomology, Poincaré duality, Poincaré-Hopf index theorem, Lefschetz fixed point theorem. To be followed in the spring by the elements of differential geometry.

Assignments: A substantial assignment will be given every other week, and graded in part. I encourage you to discuss assignment problems with your peers, subject to the following ground rules: (1) make a serious effort to think through each question for yourself first; (2) list the names of all collaborators at the head of each assignment; (3) write up all solutions in your own words; (4) do not exchange any written work with others.

Assignments will be due on Mondays at noon in my mailbox on the 2 1/2th floor of Mathematics. You must use a staple or paper clip and submit all problems together (not piecemeal). Late assignments will be penalized by 7% of their point value for each day (or part of a day) they are late.

Roundtables: There may be a roundtable discussion each week, at a time to be arranged, led by Karsten Gimre ⟨gimre@math.columbia.edu⟩. Once or twice each semester, you may be asked to make a brief presentation at the roundtable.

Exam: Final exam Monday, December 16, 4:10–7 pm (tentative). If you foresee conflicts with this date, such as a religious holiday or a language exam, you must let me know immediately. You can be excused from an exam only in a medical or family emergency, documented by a note from your doctor or dean.

Grading: Homework 50%, final 50%.

Course home page: `http://www.math.columbia.edu/~thaddeus/geometry.html`

Prerequisites from Analysis

1. **The Inverse Function Theorem**: Boothby II 6.4; Lee 7.6; Rudin, *Principles of Mathematical Analysis*, 9.17

2. **Existence and Uniqueness of Solutions to First-Order ODEs**: Lee 17.9; Rudin, Exercises 5.18, 7.23

3. **Sard's Theorem**: Guillemin & Pollack, *Differential Topology*, Appendix 1; Milnor, *Topology from the Differentiable Viewpoint*, §3