*1. For $n > 1$ and $d > 0$, prove that the general hypersurface of degree $d$ in $\mathbf{P}^n$ is irreducible.

*2. For $m \leq n$, let $Q$ be the quadric $x_0^2 + \ldots + x_m^2 = 0$ in $\mathbf{P}^n$. Show that its intersection with a general $m$-dimensional projective subspace is smooth. Hint: use the charts for the Grassmannian consisting of graphs.

*3. Let $R$ be a ring, $S$ a multiplicatively closed subset, $I$ a ideal. Let $S^{-1}I \subset S^{-1}R$ be the ideal consisting of fractions of the form $r/s, r \in I, s \in S$. Show that $S^{-1}R/S^{-1}I \cong T^{-1}(R/I)$, where $T$ is the image of $S$. 

*4. For general nonzero $f \in K[x_1, \ldots, x_n]$, describe the ring of regular functions on the open subset $A^n \backslash V(f)$ as a ring of fractions.

*5. For $x \in X$ a variety over $K$, a germ of a regular function at $x$ is a pair $(f, U)$ of an open $U \ni x$ and a regular $f : U \to K$, modulo the equivalence that $(f, U) = (g, V)$ if $f|_{U \cap V} = g|_{U \cap V}$.

Suppose $X$ is affine but possibly reducible (this is the point of the exercise). Let $R$ be its coordinate ring. If $x \in X$ has maximal ideal $m$, show that the localization $R_m$ is isomorphic to the ring of germs of regular functions at $x$.

*6. Let $M$ be a finitely generated graded module over $K[x_0, \ldots, x_n]$ with its standard grading, $N$ a graded submodule. Prove that the Hilbert polynomials satisfy $H(M, t) = H(N, t) + H(M/N, t)$.

*7. Over $R = K[x_0, \ldots, x_n]$ with its standard grading, find the Hilbert polynomials of the following modules:

(a) the shifted free module $R[m]$;
(b) $R/I$, where $I = (x_0^2 + \cdots + x_n^2)$ is the homogeneous ideal of a smooth quadric;
(c) $R/J$, where $J = (x_0^2 + x_1^2)$ is the homogeneous ideal of a singular quadric.

*8. Let $M = \bigoplus M_i$ and $N = \bigoplus N_i$ be graded modules over a graded ring $R = \bigoplus R_i$. A homomorphism of graded modules is a module homomorphism $\phi : M \to N$ such that $\phi(M_i) \subset N_i$.

(a) If $f, g \in R = K[x_0, \ldots, x_n]$ are homogeneous of degrees $d$ and $e$ respectively, show that $\phi : R[d] \oplus R[e] \to (f, g)$ given by $\phi(r, s) = rf + sg$ is a surjective homomorphism of graded modules.

(b) To what familiar graded module is $\ker \phi$ isomorphic when $f$ and $g$ are coprime? What about for general $f, g$?

(c) Compute the Hilbert polynomial of $R/(f, g)$.

(d) Discuss the geometric meaning when $n = 2$. 