1. Must a module of finite length over a local ring $R$ be a vector space over $R/m$? Give a proof or counterexample.

2. State and prove a characterization of $V(\text{Ann } M)$ for $M$ a module over $k[x_1, \ldots, x_n]$ of finite length, where $k$ is algebraically closed as usual.

3. If $\phi_d : \mathbb{P}^n \to \mathbb{P}^N$ is the degree $d$ Veronese embedding, then the degree of $\phi_d(\mathbb{P}^n)$ is $d^n$.

4. (a) The degree of the Segre embedding of $\mathbb{P}^r \times \mathbb{P}^s$ in $\mathbb{P}^n$ is $\binom{r+s}{r}$.
   (b) Generalize to $\mathbb{P}^{r_1} \times \cdots \times \mathbb{P}^{r_m}$.

5. For an $r$-dimensional variety $X \subset \mathbb{P}^n$ with Hilbert polynomial $P_X$, define the arithmetic genus to be $p_a(X) := (-1)^r(P_X(0) - 1)$. Hartshorne: “This is an important invariant which, as we will see later, is independent of the projective embedding of $Y$.”
   (a) First of all, $p_a(\mathbb{P}^n) = 0$.
   (b) If $Y$ is a plane curve of degree $d$, then $p_a(Y) = (d - 1)(d - 2)/2$.
   (c) More generally, if $H$ is a hypersurface in $\mathbb{P}^n$ of degree $d$, then $p_a(H) = \binom{d-1}{n}$.
   (d) If $Y \subset \mathbb{P}^r$, $Z \subset \mathbb{P}^s$ are of dimensions $d$, $e$ respectively, and $Y \times Z \subset \mathbb{P}^n$ is embedded via the Segre embedding $\mathbb{P}^r \times \mathbb{P}^s \to \mathbb{P}^n$, then

   $$p_a(Y \times Z) = p_a(Y)p_a(Z) + (-1)^e p_a(Y) + (-1)^d p_a(Z).$$

6. If a triangle is inscribed in a smooth conic, then the intersections of its sides with the tangents to the conic at the opposite vertices are collinear.