1. Every constructible set contains a dense open subset of its closure.

2. In a graded ring, a homogeneous ideal \( p \) is prime if and only if it satisfies the condition that \( ab \in p \) implies \( a \in p \) or \( b \in p \) for homogeneous \( a, b \) only.

3. Every irreducible curve (over an algebraically closed field) is birational to a plane curve.

4. (a) If \( P^N = Pk[x_0, \ldots, x_m]_d \) is the projective space parametrizing homogeneous polynomials of degree \( d \) in \( x_0, \ldots, x_m \), then there exists a dense open \( U \subset P^N \) such that \( V(f) \subset P^m \) is smooth for every \([f] \in U\).

(b) If instead \( P^N \) parametrizes bihomogeneous polynomials in \( x_0, \ldots, x_m \) and \( y_0, \ldots, y_n \) of bidegree \((d, e)\), then there again exists a dense open \( U \subset P^N \) such that \( V(f) \subset P^m \times P^n \) is smooth for every \([f] \in U\).

5. Over a PID, a finitely generated module has finite length if and only if it contains no free submodule. (You may use the classification of finitely generated modules over a PID, which is exactly parallel to the classification of finitely generated abelian groups.)

6. (a) Over \( \mathbb{C}[t] \), let \( M = \mathbb{C}^2 \), where \( t \) acts by multiplication by a fixed \( 2 \times 2 \) matrix \( A \). What is the length \( \ell(M) \) in terms of the entries of \( A \)?

(b) Same thing with \( \mathbb{C} \) replaced by \( \mathbb{R} \).

7. Over an algebraically closed field, every linear subspace of dimension \( > (m-1)(n-1) \) of the space of \( m \times n \) matrices contains a matrix of rank 1.