1. Let $V$ be a finite-dimensional vector space, and let $\text{Seg} : \mathbb{P}V \times \mathbb{P}V \to \mathbb{P}(V \otimes V)$ be the Segre embedding. The eigenspaces of the natural involution on $V \otimes V$ decompose it as $V \otimes V = \text{Sym}^2 V \oplus \Lambda^2 V$. Describe, with proof, the intersection of $\text{Seg} (\mathbb{P}V \times \mathbb{P}V)$ with $\mathbb{P} \text{Sym}^2 V$ and with $\mathbb{P} \Lambda^2 V$.

2. The image of the Plücker embedding $\text{Pl} : \text{Gr}_2 \mathbb{A}^4 \to \mathbb{P}^5$ is a smooth quadric hypersurface.
   
   (a) Write down the equation of this hypersurface explicitly.
   
   (b) Describe the image of the locus $X_v$ where the subspace $V \subset \text{Gr}_2 \mathbb{A}^4$ contains a fixed nonzero $v \in \mathbb{A}^4$, and also the image of the locus $X_f$ where the subspace $V$ is annihilated by a fixed nonzero linear functional $f : \mathbb{A}^4 \to k$.

In the remaining problems, $V$ and $W$ denote varieties over $k$.

3. If $f : V \to W$ is any morphism, then $\text{id}_V \times f : V \to V \times W$ is a closed embedding. (Call its image the \textit{graph} of $f$ and denote it by $\Gamma(f)$.)

4. (a) If $f : V \to W$ is any morphism with $V$ complete, then $f(V)$ is closed in $W$ and complete.
   
   (b) Give a counterexample when $W$ is a nonseparated prevariety.

5. If an open subset $U$ of a variety $V$ is complete, then $U = V$ or $U = \emptyset$.

6. If $f \in k[V]$ is a regular function, then its graph is closed even as a subset of $V \times \mathbb{P}^1 \supset V \times \mathbb{A}^1$.

7. If $V$ is complete, then $k[V] = k$.

8. If an affine variety is complete, then it is a point.