1. Compute the cohomology of the skyscraper sheaf $\mathcal{O}_x$ of a point $x \in X$.

2. We proved that if a line bundle $L$ over a smooth curve $C$ has degree $\deg L \geq 0$, then $\dim H^0(C, L) \leq \deg L + 1$. One might conjecture that if a vector bundle $E$ over $C$ has degree $\deg E \geq 0$, then $\dim H^0(C, E) \leq \deg E + \text{rank} E$. Is this true? Proof or counterexample.

3. (a) Let $\phi : \mathbb{P}^1 \to \mathbb{P}^1$ be $z \mapsto z^2$. Prove that $\phi_* \mathcal{O}(d) \cong \mathcal{O}(d+) \oplus \mathcal{O}(d_-)$ for certain $d_+$, $d_-$, and determine them as functions of $d$.

   (b) Optional if you feel energetic: do likewise for $z \mapsto z^n$.

4. Let $S$ over $\mathbb{A}^1$ be the extension by zero of $\mathcal{O}_{\mathbb{A}^1 \setminus \{0\}}$, that is, $U \mapsto \mathcal{O}(U)$ if $0 \notin U$, but $U \mapsto 0$ if $0 \in U$.

   (a) Show that $H^1(\mathbb{A}^1, S)$ is infinite-dimensional. Hint: use a long exact sequence.

   (b) Why does this not contradict Serre’s theorem?

5. If $0 \to S' \to S \to S'' \to 0$ is an exact sequence of sheaves over $X$, with $S$ flabby, and $\phi : X \to Y$ is continuous, then $0 \to \phi_* S' \to \phi_* S \to \phi_* S'' \to 0$ is exact over $Y$.

6. If $F$ is flabby over $X$ and $\phi : X \to Y$ is continuous, then $R^i \phi_* F = 0$ for all $i > 0$.

7. If $0 \to S' \to S \to S'' \to 0$ is an exact sequence of sheaves over $X$, and $\phi : X \to Y$ is continuous, then the higher direct images of $S'$, $S$, and $S''$ form a long exact sequence.

8. (a) Let $0 \to F \to F^0 \to F^1 \to \cdots$ be any exact sequence of sheaves on $X$, $\phi : X \to Y$ a continuous map such that $R^i \phi_* F^j = 0$ for $i > 0$. Show that the cohomology of the complex $0 \to \phi_* F^0 \to \phi_* F^1 \to \cdots$ is $R^i \phi_* F$.

   (b) Show that the higher direct images may be computed with any flabby resolution.