1. Prove that no affine curve is complete.

2. Use the previous problem to prove that no affine variety (of dim > 0) is complete.

3. Use the previous problem to prove that any affine proper morphism has finite fibers.

4. Let $X$ be $wz - xy = 0$, the cone in $\mathbb{A}^4$ on a smooth quadric surface in $\mathbb{P}^3$. Let $A$, $B$ be the planes $x = z = 0$ and $y = z = 0$ respectively. Show that (1) the closure of $X \setminus A$ in $\text{Bl}(\mathbb{A}^4, A)$; (2) the closure of $X \setminus B$ in $\text{Bl}(\mathbb{A}^4, B)$; and (3) the closure of $X \setminus 0$ in $\text{Bl}(\mathbb{A}^4, 0)$ are all resolutions of $X$ and are all distinct in the sense that there are no isomorphisms between them preserving the morphisms to $X$ (though there is an isomorphism between the first two not preserving these morphisms).

5. For $f : X \rightarrow Y$ a morphism of schemes, $x \in X$, define a derivative $D_x f : T_x X \rightarrow T_{f(x)} Y$, linear over the residue field $k(f(x))$ of $f(x)$, and show that it satisfies the chain rule.

6. Let $X$ be a scheme, $x \in X$ a point with residue field $k$ and Zariski tangent space $T_x X = (\mathfrak{m}/\mathfrak{m}^2)^*$, a vector space over $k$. Let the ring of dual numbers be $D = k[\epsilon]/(\epsilon^2)$. Show that for every $v \in T_x X$ there is an unique morphism $\text{Spec } D \rightarrow X$ with image $x$ and derivative taking 1 to $v$. Interpret addition $v_1 + v_2$ and scalar multiplication $tv$ in terms of the geometry of $\text{Spec } D$.

7. (a) Show that every constructible set in a variety contains an open dense subset of its closure.

(b) An algebraic group is a variety $G$ with a group structure such that the group operations are morphisms. An algebraic group homomorphism is a homomorphism of algebraic groups which is also a morphism. Prove that every algebraic group homomorphism has closed image.