

Mathematics GU4042
Introduction to Modern Algebra II
Spring 2017

Instructor: Prof. Michael Thaddeus **Classroom:** Hamilton 602
Office: Mathematics 414 **Lectures:** T.Th. 4:10–5:25 pm
Office hours: F. 10:30 am – 12:30 pm, or by appointment.

Prerequisites: Introduction to Modern Algebra I or the instructor’s explicit permission.

Recommended texts: *Abstract Algebra* by David S. Dummit and Richard M. Foote (Wiley), or *A First Course in Abstract Algebra* by John B. Fraleigh (Addison Wesley Longman).

Course description: We have already studied the theory of *abelian groups*, which could roughly be described as number systems having $+$ and $-$. We now proceed to the study of *rings*, which could likewise be described as systems having $+$, $-$, and \times , and *fields*, systems having $+$, $-$, \times , and \div . The obvious examples are \mathbf{Z} and \mathbf{Q} respectively, but what give the subject its beauty and power are the many non-obvious examples. The operations in a ring allow us to define *polynomials*, and the study of rings will teach us much about them. The course will culminate in the famous theorem of Abel-Ruffini-Galois asserting, roughly, that there is no formula in terms of radical signs (like the quadratic formula) for the roots of a general polynomial of degree ≥ 5 .

Course outline: Rings and fields. Homomorphisms and ideals. Integral domains, Euclidean domains, principal ideal domains, unique factorization domains, and the relations between them. Fraction fields. Unique factorization for polynomial rings. Expressions for prime numbers as sums of squares. The Galois group of a field extension. Ruler and compass constructions. Finite fields and their classification. Finite, separable, and normal extensions. The fundamental theorem of Galois theory. Insolvability of the quintic by radicals.

Assignments: To learn a subject like this one thoroughly, practical experience is essential, so a written assignment will be given each week. It will be posted on the course home page. You can learn as much from your fellow students as from lectures, so I encourage you to discuss the problems with each other, subject to the following ground rules: (1) make a serious effort to think through each question for yourself first; (2) list the names of all collaborators at the head of each assignment; (3) do not exchange any written work with others; (4) do not consult any Internet sources except Wikipedia; (5) write up every problem in your own words.

On each assignment, the use of a staple or paper clip is absolutely, positively, utterly compulsory! Also, each assignment must be submitted as a single document, not piecemeal. Addenda to your assignment will not be graded. And, unfortunately, electronic submission of assignments (e.g. by fax or e-mail) cannot be accepted. Only a physical copy in the collection box will do. If you must be out of town, please ask a fellow student to print out and submit your assignment.

Assignments are due on Fridays at 5 pm in the mailbox outside 417 Mathematics marked “Modern Algebra II.” Late assignments will be penalized by 10% of their point value for each day they are late. Warning: the building may be locked outside of library hours.

Exams: There will be two midterm exams in class Thursday, March 2 and Thursday, April 13. There will not normally be makeup exams for the midterms; instead, you may be given an oral exam covering the same material. The final exam is tentatively scheduled for Tuesday, May 9, 4:10–7 pm. If you foresee conflicts, such as a religious holiday, with any of these exams let me know immediately. You can be excused from an exam only in a medical or family emergency, documented by a note from your doctor or dean. Also, please make your travel plans for the summer break early, as the date of the final exam cannot be moved.

Grading: 1/5 assignments, 1/5 each midterm, 2/5 final.

Devices: Cellphones and other electronic devices must be turned *off* and put *away* during class. Many thanks!

Help Room: You may wish to take advantage of the Mathematics Help Room in 406 Mathematics. Teaching assistants are on call for much of the week to help you with any math problems you may experience. Schedules will be posted shortly at:
<http://www.math.columbia.edu/programs/main/one/helprooms.html>

Contacting me: By e-mail at thaddeus@math.columbia.edu, or by telephone at 4–4308. Preferably, come to my office hours on Friday from 10:30 am to 12:30 pm in 414 Mathematics, or knock on my door at any time.

Course home page: <http://www.math.columbia.edu/~thaddeus/modernalgebra2017.html>