1. Define (a) a separable polynomial; (b) a separable extension.

2. Is the real number $3^{1/7}$ constructible with ruler and compass? What about $3^{1/8}$? Why?

3. If $E = \mathbb{Q}(r)$ where $r$ has minimal polynomial of prime degree over $\mathbb{Q}$, prove that there are no fields between $E$ and $\mathbb{Q}$.

4. Must the splitting field of any polynomial over a finite field be a finite field? Why?

5. Prove or disprove: the rings $\mathbb{F}_2[x]/(x^3 + x + 1)$ and $\mathbb{F}_2[x]/(x^3 + x^2 + 1)$ are isomorphic.

6. Let $E \subset \mathbb{C}$ be a subfield such that $\tau \in E$ whenever $z \in E$. Let $F = E \cap \mathbb{R}$. Prove that $E/F$ is a normal extension.

7. Describe, sketching proofs, the Galois groups of the splitting fields $E$ over $\mathbb{Q}$ of:
   (a) $x^5 + 1$
   (b) $x^3 + 2$
   (c) $x^4 - 9$