# Mathematics GU4042
# Introduction to Modern Algebra II

## Practice Midterm #2

April 13, 2017

**1.** Define (a) a separable polynomial; (b) a separable extension.

**2.** Is the real number $3^{1/7}$ constructible with ruler and compass? What about $3^{1/8}$? Why?

**3.** If $E = \mathbb{Q}(r)$ where $r$ has minimal polynomial of prime degree over $\mathbb{Q}$, prove that there are no fields between $E$ and $\mathbb{Q}$.

**4.** Must the splitting field of any polynomial over a finite field be a finite field? Why?

**5.** Prove or disprove: the rings $\mathbb{F}_2[x]/(x^3+x+1)$ and $\mathbb{F}_2[x]/(x^3+x^2+1)$ are isomorphic.

**6.** Let $E \subset \mathbb{C}$ be a subfield such that $\overline{z} \in E$ whenever $z \in E$. Let $F = E \cap \mathbb{R}$. Prove that $E/F$ is a normal extension.

**7.** Describe, sketching proofs, the Galois groups of the splitting fields $E$ over $\mathbb{Q}$ of:

(a) $x^5 + 1$

(b) $x^3 + 2$

(c) $x^4 - 9$