Mathematics GU4042
Introduction to Modern Algebra II

Practice Midterm #1

March 2, 2017

Attempt all 7 problems. Each is worth 10 points. Good luck!

1. A theorem from class states that a ring R is a ufd if and only if two conditions (a) and (b) are true. What are they?

2. Prove the converse of Gauss's Lemma: if $fg = h \in \mathbb{Z}[x]$ is primitive, then f and g are primitive.

3. (a) Proof or counterexample: $\phi : R \to S$ a surjective homomorphism and R a domain imply S a domain.

(b) Proof or counterexample: $\phi : R \to S$ a surjective homomorphism, R, S both domains, and R a pid imply S a pid.

4. Which of the following are irreducible in $\mathbb{Z}[x]$ and why? (a) $x^4 + 4x^3 + 6x^2 + 4x + 1$; (b) $x^4 + 4x^3 + 6x^2 + 4x + 2$; (c) $x^4 + x^3 + x^2 + x + 1$; (d) $x^4 + x^3 + x^2 + x$.

5. Prove that if D is an integral domain, then so is $D[x]$ (for x an indeterminate).

6. If p is a prime number and $a \in \mathbb{F}_p$, show that $x^p - a$ is always reducible in $\mathbb{F}_p[x]$.

7. In a ufd R, suppose that a, b, c are nonzero nonunits such that $a \mid bc$ and 1 is a gcd of a and b. Prove that $a \mid c$.