

Mathematics GU4041
Introduction to Modern Algebra I
Fall 2016

Instructor: Prof. Michael Thaddeus **Classroom:** Mathematics 312
Office: Mathematics 414 **Lectures:** T.Th. 4:10–5:25 pm
Office hours: F. 10:30 am–12:30 pm, or by appointment.

Prerequisites: Calculus IV and Linear Algebra, or Honors Math A–B, or the instructor’s explicit permission.

Optional texts: *A First Course in Abstract Algebra* by John B. Fraleigh (Addison-Wesley); *Abstract Algebra* by David S. Dummit and Richard M. Foote (Wiley), both on reserve in the Mathematics Library.

Course description: Algebra was created in the Arab world in medieval times, but it took its present shape in the hands of such 19th-century masters as Cauchy, Abel, Galois, and Cayley. We will study the theory of groups as they developed it. It is in some sense an abstract formulation of the intuitive notion of symmetry. Our twin goals will be to learn the subject matter and the language of rigorous proof in which it is formulated. The course continues with the study of rings and fields in the spring, culminating in Galois’s celebrated proof that most polynomial equations of degree five or higher cannot be solved by radicals (that is, with a formula involving roots like the quadratic formula).

Course outline: Preliminary material on propositional and predicate logic, the algebra of sets and functions, equivalence relations, and the integers and rational numbers. Groups, homomorphisms, group actions. The isomorphism theorems. The Jordan-Hölder theorem. Lagrange’s, Cayley’s, and Sylow’s theorems on finite groups. Burnside’s lemma and the enumeration of orbits. Direct products, semidirect products, free groups. Classification of finitely generated abelian groups.

Assignments: To learn a subject like this one thoroughly, practical experience is essential, so a written assignment will be given each week. It will be posted on Courseworks. You can learn as much from your fellow students as from lectures, so I encourage you to discuss the problems with each other, subject to the following ground rules: (1) do not consult any online sources except Wikipedia; (2) make a serious effort to think through each question for yourself first; (3) list the names of all collaborators at the head of each assignment; (4) do not exchange any written work with others; (5) write up every problem in your own words.

Assignments are due on Mondays at 5 pm, in the Modern Algebra mailbox outside 417 Mathematics. Online submission is not allowed; only a hard copy in the submission box is acceptable. The use of a staple or paper clip and the submission of all problems together (not piecemeal) is absolutely, positively, utterly compulsory. Late assignments will be penalized by 10% of their point value for each day they are late. Warning: the building may be locked outside of library hours.

Exams: There will be two midterm exams in class Thursday, October 6 and Thursday, November 17. There will not normally be makeup exams for the midterms; instead, you may be given an oral exam covering the same material. The final exam is tentatively scheduled for Tuesday, December 20, 4:10–7 pm. If you foresee conflicts, such as a religious holiday, with any exam let me know immediately. You can be excused from an exam only in a medical or family emergency, documented by a note from your doctor or dean. Also, please make your travel plans for the winter break early, as the date of the final exam cannot be moved.

Grading: 1/5 assignments, 1/5 each midterm, 2/5 final.

Devices: Phones, laptops, tablets, and all other electronic devices (except watches) must be turned off and put away during all classes and exams.

Help Room: You may wish to take advantage of the Mathematics Help Room in 406 Mathematics. Teaching assistants are on call for much of the week to help you with any math problems you may experience. Schedules will be posted shortly: see www.math.columbia.edu/programs/main/one/helprooms.html.

Contacting me: By e-mail at mt324@columbia.edu, or by telephone at 212–854–4308. Preferably, come to my office hours on Friday from 10:30 am to 12:30 pm in 414 Mathematics, or knock on my door at any time.

Course assistants: Our course assistants, Mitchell Faulk (faulk@math.columbia.edu) and John Long (jj12215@columbia.edu), will both have hours in the Help Room.