Mathematics GU4041
Introduction to Modern Algebra

Practice Midterm Exam #2

November 17, 2016

READ AND FOLLOW CAREFULLY ALL INSTRUCTIONS BELOW

Turn off all electronic devices.
Brief visits to the men's or women's room (just out the door to your left) are OK, but one at a time only, and you must ask permission first.
Write your name, "Modern Algebra, Prof. Thaddeus," and the number of blue books on the cover of each blue book.
Write your name on the attendance sheet when it comes around.
Write all answers and work in your blue books. Do not hand in this sheet.
On each page you use, write the number of the problem *in a circle in the margin.*
You may do more than one problem on a single page. Just put all numbers by their problems.
You may do the problems out of order, but this is discouraged as it can lead to misgrading.
When there is any doubt, state briefly but clearly what statements from the text, lecture, or assignments you are using.
In grading the exams, I will emphasize accuracy, brevity, and clarity. Aim for all three.
Attempt all 7 problems. Each is worth 10 points. Good luck!

1. State the second isomorphism theorem.

2. Let G be a group whose only subgroups are itself and 1. Prove that $G \cong 1$ or $G \cong \mathbb{Z}_p$ for some prime p.

3. Let N_1, N_2 be normal subgroups of G. Prove that the subgroup $N_1 \cap N_2$ is also normal in G.

4. Let G and H be finite groups whose orders are relatively prime. Prove that the only homomorphism $\phi : G \to H$ is $\phi(g) = e$.

5. Recall that subgroups $P, Q < G$ are *conjugate* if $Q = gPg^{-1}$ for some $g \in G$. Let G act on a set S, and suppose $x, y \in S$ belong to the same orbit. Prove that the stabilizer groups G_x and G_y are conjugate.

6. Let $\phi : G_1 \times G_2 \to H$ be a homomorphism. Prove that there are homomorphisms $\phi_1 : G_1 \to H$ and $\phi_2 : G_2 \to H$ such that for all $(g_1, g_2) \in G_1 \times G_2$, $\phi(g_1, g_2) = \phi_1(g_1)\phi_2(g_2)$, and also $\phi(g_1, g_2) = \phi_2(g_2)\phi_1(g_1)$.

7. Let $G = \Sigma_n$ be the symmetric group on n elements, acting on itself by conjugation, and let $g = (12)$ be the transposition of the first two elements. What is $\#G_g$, the order of its stabilizer? Prove your answer correct.