Mathematics GU4041
Introduction to Modern Algebra

Practice Midterm #1

October 6, 2016

Attempt all 7 problems. Each is worth 10 points. Good luck!

1. State the definition of an equivalence relation.

2. Prove that for any sets A, B, C, we have $A \setminus (B \cup C) = (A \setminus B) \setminus C$.

3. Let $\mathbb{Z}_5$ denote the set of integers modulo 5. Let $[0]$ be the zero element, and let $f([n]) := [n+1]$ be the successor function. Which of the Peano axioms does $\mathbb{Z}_5$ then satisfy, and which does it violate? For each axiom, prove your answer correct.

4. For natural numbers k, m, n with $m \neq 0 \neq n$, show that the function $f : \mathbb{Z}_m \to \mathbb{Z}_n$ given by $f([i]) := [ki]$ is well-defined if and only if $n \mid km$.

5. Prove that for all $m, n \in \mathbb{N}$, either $m + n \neq m$ or $n = 0$.

6. Prove that if m, n, p, q are natural numbers with $m \mid n$ and $p \mid q$, then $mp \mid nq$.

7. Let (x, y) denote the greatest common divisor of $x, y \in \mathbb{Z}$. Prove that $(x, y) = (x+y, y)$.