1. There exist a set \( \mathbb{N} \), an element \( 0 \in \mathbb{N} \), and a successor function \( f : \mathbb{N} \to \mathbb{N} \), denoted \( f(n) = n' \), satisfying the following axioms.

P1: There does not exist \( n \in \mathbb{N} \) such that \( 0 = n' \).

P2: For all \( m, n \in \mathbb{N} \), if \( m' = n' \), then \( m = n \).

P3: If \( S \subset \mathbb{N} \) is a subset, if \( 0 \in S \), and if all \( n \in S \) satisfy \( n' \in S \), then \( S = \mathbb{N} \).

2. Note \( x \in X \setminus (A \cup B) \iff x \in X \) but \( x \not\in A \cup B \iff x \in X \) but it is false that \( x \in A \) or \( x \in B \iff x \in X \) and \( x \not\in A \) and \( x \not\in B \iff x \in X \) and \( x \not\in A \) and \( x \not\in X \) and \( x \not\in B \iff x \in X \setminus A \) and \( x \in X \setminus B \iff x \in (X \setminus A) \cap (X \setminus B) \). Hence the two sides are equal by the definition of set equality.

3. If \( g \circ f(x) = g \circ f(y) \), then \( g(f(x)) = g(f(y)) \) (by definition of \( \circ \)), hence \( f(x) = f(y) \) (since \( g \) is injective), hence \( x = y \) (since \( f \) is injective). Hence \( g \circ f \) is injective.

Given \( z \in U \), there exists \( y \in T \) such that \( z = g(y) \) (since \( g \) is surjective), and there exists \( x \in S \) such that \( y = f(x) \) (since \( f \) is surjective). Then \( x = g(f(x)) = g \circ f(x) \), so \( g \circ f \) is surjective.

Alternative proof: By the main theorem on inverses, there exist functions \( f^{-1} : T \to S \) and \( g^{-1} : U \to T \) such that \( f^{-1} \circ f = \text{id}_S \), \( f \circ f^{-1} = \text{id}_T \), \( g^{-1} \circ g = \text{id}_U \), and \( g \circ g^{-1} = \text{id}_U \).

Then \( (g \circ f) \circ (f^{-1} \circ g^{-1}) = g \circ (f \circ f^{-1}) \circ g^{-1} = g \circ (\text{id}_T) \circ g^{-1} = g \circ g^{-1} = \text{id}_U \) and \( (f^{-1} \circ g^{-1}) \circ (g \circ f) = f^{-1} \circ (g^{-1} \circ g) \circ f = f^{-1} \circ (\text{id}_T) \circ f = f^{-1} \circ f = \text{id}_S \), so \( f^{-1} \circ g^{-1} \) is an inverse for \( g \circ f \). By the main theorem on inverses again, \( g \circ f \) is bijective.

4. If \( m \leq n \), then by definition of \( \leq \) there exists \( a \in \mathbb{N} \) such that \( n = m + a \). Then \( n^2 = n \cdot n = (m + a) \cdot (m + a) = (m + a) \cdot m + (m + a) \cdot a = (m^2 + am) + (ma + a^2) = m^2 + (am + ma + a^2) \). Since \( \mathbb{N} \) is closed under addition and multiplication by the definitions of these operations, \( am + ma + a^2 \in \mathbb{N} \), so \( m^2 \leq n^2 \) by the definition of \( \leq \).

5. Yes: it is reflexive since \( x - x = 0 = x \) if \( x \in \mathbb{Z} \), symmetric since \( x - y = -(x - y) = x \) if \( x \in \mathbb{Z} \) implies \( y - x = -(x - y) = z \) if \( z \in \mathbb{Z} \), and transitive since \( x - y = z \) and \( y - z = w \) imply \( x - w = x - y + y - z = y - z + x - y = (x - y) + (y - z) = z \) if \( z \in \mathbb{Z} \).

6. Since \( a \mid b \), there exists \( m \in \mathbb{Z} \) such that \( b = am \), and since \( b \mid c \), there exists \( n \in \mathbb{Z} \) such that \( c = bn \). Then \( a + b + c = a + am + bn = a + am + (am)n = a(1 + m + mn) \) where \( 1 + m + mn \in \mathbb{Z} \) since \( \mathbb{Z} \) is closed under addition and multiplication. Hence \( a \mid a + b + c \).

7. By A4\#4, \( (i, n) = 1 \) if and only if \( [i] \in \mathbb{Z}_n \) has a reciprocal. Hence there exist \( [c], [d] \in \mathbb{Z}_n \) such that \( [ac] = [a][c] = [1] \) and \( [bd] = [b][d] = [1] \) in \( \mathbb{Z}_n \). Hence \( [ab][cd] = [abcd] = [ac][bd] = [1][1] = [1] \in \mathbb{Z}_n \), so \( [ab] \) also has a reciprocal and hence \( (ab, n) = 1 \).