# Mathematics GU4041
# Introduction to Modern Algebra

## Midterm Examination #1

October 6, 2016

### READ AND FOLLOW CAREFULLY ALL INSTRUCTIONS BELOW

Turn off all electronic devices.
Brief visits to the men's or women's room (up or down one flight) are OK, but one at a time only, and leave your phone at the front.
Write your name, "Modern Algebra, Prof. Thaddeus," and the number of blue books on the cover of each blue book.
Write your name on the attendance sheet when it comes around.
Write all answers and work in your blue books. Do not hand in this sheet.
On each page you use, write the number of the problem *in a circle in the margin.*
You may do more than one problem on a single page. Just put all numbers by their problems.
You may do the problems out of order, but this is discouraged as it can lead to misgrading.
When there is any doubt, state briefly but clearly what statements from the text, lecture, or assignments you are using.
In grading the exams, I will emphasize accuracy, brevity, and clarity. Aim for all three.
Attempt all 7 problems. Each is worth 10 points. Good luck!

**1.** State, in complete grammatical sentences, the Peano Axioms for the natural numbers.

**2.** If $A$ and $B$ are subsets of $X$, prove that $X \setminus (A \cup B) = (X \setminus A) \cap (X \setminus B)$. (It may help to draw a Venn diagram first.)

**3.** Let $f : S \to T$ and $g : T \to U$ be functions. Prove that if $f$ and $g$ are bijective, then $g \circ f$ is bijective. (Recall that *bijective* means injective and surjective.)

**4.** Prove that for all natural numbers $m, n \in \mathbf{N}$, if $m \leq n$, then $m^2 \leq n^2$. (Here $m^2 = m \cdot m$ and $n^2 = n \cdot n$.)

**5.** On the set $\mathbf{Q}$ of rational numbers, define a relation $\sim$ by: $x \sim y$ if and only if $x - y \in \mathbf{Z}$. Is this an equivalence relation? Explain thoroughly why or why not.

**6.** Prove for all integers $a, b, c \in \mathbf{Z}$ that $a \mid b$ and $b \mid c$ imply $a \mid (a + b + c)$.

**7.** Prove for all $a, b, n \in \mathbf{Z}$ with $n > 0$ that $(a, n) = 1$ and $(b, n) = 1$ imply $(ab, n) = 1$. (Here $(a, n)$ denotes the greatest common divisor.)