1. Let $S$ be a set and $G$ a group. Denote by $F(S,G)$ the set of all functions $f : S \to G$. Define a binary operation on $F(S,G)$ and prove that it is a group. Interpret $F(S,G)$ in the case $S = \mathbb{R}, G = \mathbb{R}$.

2. (a) Recall that the symmetric group $\Sigma_n$ is the group of all bijections $f : \langle n \rangle \to \langle n \rangle$, where $\langle n \rangle = \{1, \ldots, n\}$. Prove that $\{ f \in \Sigma_n \mid f(1) = 1 \}$ is a subgroup of $\Sigma_n$.

(b) Let $S$ be any set and let $x \in S$. Recall that the set Bij $S$ of all bijections $S \to S$ is a group (with $\circ$). Prove that $\{ f \in \text{Bij} S \mid f(x) = x \}$ is a subgroup of Bij $S$.

3. It was proved in class that all subgroups of $\mathbb{Z}$ (with $+$) are of the form $k\mathbb{Z}$, where $k \in \mathbb{N}$. State and prove a similar description of all subgroups of $\mathbb{Z} \times \mathbb{Z}_2$. Hint: draw some pictures first.

4. If $G$ is a finite group and $\#G$ is even, prove that $G$ contains an element besides the identity which equals its own inverse. (Hint: consider $\{ g \in G \mid g \neq g^{-1} \}$; how many elements does it have?) What is this element for $G = \mathbb{Z}_2^n$?

5. Let $g$ and $h$ be elements of a group $G$ with identity $e$. Prove that $gh = hg$ if and only if $h^{-1}gh = g$ if and only if $g^{-1}h^{-1}gh = e$.

6. Prove that if $G$ is a group, and if all $g \in G$ satisfy $g^2 = e$, then $G$ is abelian.

7. Let $G$ and $H$ be groups. Prove that $G \times H$ is abelian if and only $G$ and $H$ are both abelian.