# Mathematics GU4041
# Introduction to Modern Algebra

**Assignment #13**

Due December 7, 2016

**1.** List all 12 elements of $A_4$. Show that $A_4$ is not simple by exhibiting a proper normal subgroup. Hint: look back at the classification of groups of order 12.

**2.** (a) For $n \geq 3$, show that $\Sigma_n$ has no normal subgroup of order 2.
(Careful, it says order 2, not index 2!)

(b) For $n \geq 5$, show that $A_n$ and 1 are the only proper normal subgroups of $\Sigma_n$.
Hint: use the Second Isomorphism Theorem.

**3.** (a) Show that the center of a direct product is the direct product of the centers:

$$Z(G_1 \times \cdots \times G_n) = ZG_1 \times \cdots \times ZG_n.$$

(b) Show that $G_1 \times \cdots \times G_n$ is abelian if and only if each $G_i$ is.

**4.** (a) For $A$ an abelian group and $n \in \mathbf{N}$, show that the set $A(n)$ of all elements whose order is finite and divides $n$ is a subgroup.

(b) If $A \cong B$, show that $A(n) \cong B(n)$.

(c) Show that $\mathbf{Z}_2 \times \mathbf{Z}_2 \times \mathbf{Z}_4 \ncong \mathbf{Z}_4 \times \mathbf{Z}_4$.

**5.** Show, using the classification of finite abelian groups, that there are exactly 10 isomorphism classes of abelian groups of order 400. Hint: what are the possible Sylow subgroups?

**6.** Show, using the classification of finite abelian groups, that any finite abelian group is either cyclic or contains a subgroup isomorphic to $\mathbf{Z}_p \times \mathbf{Z}_p$ for some prime $p$.