Mathematics W4041x
Introduction to Modern Algebra

Practice Midterm #2

November 18, 2010

1. State the third isomorphism theorem.

2. Prove that every group of prime order is cyclic.

3. Prove that the subset H of matrices in $G = GL(n, \mathbf{Q})$ whose determinant is positive is a normal subgroup, and describe the quotient group G/H.

4. Give a composition series for D_{12}, the symmetry group of a regular hexagon (and prove that it is a composition series).

5. (a) Prove that the quaternion group Q is isomorphic to a subgroup of the symmetric group Σ_8.

(b) Prove, however, that Q is not isomorphic to a subgroup of Σ_n for any $n < 8$. (Hint: show that if Q acts on any set with < 8 elements, then -1 must act trivially.)

6. Prove that if G is simple and $\phi : G \to \Sigma_n$ is a homomorphism, then either $G \cong \mathbb{Z}_2$ or $\phi(G) \subset A_n$. Here Σ_n is the symmetric group, A_n the alternating group.

7. A company manufactures 3×3 tiles marked with the letters A and B. They want all periodic patterns (with period 3) to be constructible from their tiles. (See the example below.) However, they do not have to make all 2^9 possibilities: the seams between the tiles are barely visible, so the two configurations shown are equivalent. How many different types of tile do they have to keep in stock?

A	B	B	A	B	B	A	B	B
A	B	A	A	B	A	A	B	A
B	B	B	B	B	B	B	B	B
A	B	B	A	B	B	A	B	B
A	B	A	A	B	A	A	B	A
B	B	B	B	B	B	B	B	B
A	B	B	A	B	B	A	B	B
A	B	A	A	B	A	A	B	A
B	B	B	B	B	B	B	B	B

A	B	B	A	B	B	A	B	B
A	B	A	A	B	A	A	B	A
B	B	B	B	B	B	B	B	B
A	B	B	A	B	B	A	B	B
A	B	A	A	B	A	A	B	A
B	B	B	B	B	B	B	B	B
A	B	B	A	B	B	A	B	B
A	B	A	A	B	A	A	B	A
B	B	B	B	B	B	B	B	B