1. State the second isomorphism theorem.

2. Let $G$ be a group whose only subgroups are itself and 1. Prove that $G \cong 1$ or $G \cong \mathbb{Z}_p$ for some prime $p$.

3. Let $N_1, N_2$ be normal subgroups of $G$. Prove that the subgroup $N_1 \cap N_2$ is also normal in $G$.

4. Let $G$ and $H$ be finite groups whose orders are relatively prime. Prove that the only homomorphism $\phi : G \to H$ is $\phi(g) = e$.

5. Recall that subgroups $P, Q < G$ are conjugate if $Q = gPg^{-1}$ for some $g \in G$. Let $G$ act on a set $S$, and suppose $x, y \in S$ belong to the same orbit. Prove that the stabilizer groups $G_x$ and $G_y$ are conjugate.

6. Let $\phi : G_1 \times G_2 \to H$ be a homomorphism. Prove that there are homomorphisms $\phi_1 : G_1 \to H$ and $\phi_2 : G_2 \to H$ such that for all $(g_1, g_2) \in G_1 \times G_2$, $\phi(g_1, g_2) = \phi_1(g_1)\phi_2(g_2)$, and also $\phi(g_1, g_2) = \phi_2(g_2)\phi_1(g_1)$.

7. How many different necklaces can be made by stringing together seven black or white beads as shown? If the necklace is rotated or flipped over, it remains the same necklace, of course.