1. State the Axiom of Specification.

2. If $f : S \to T$ is a function, $U, V \subset T$, prove that $f^{-1}(U \cap V) = f^{-1}(U) \cap f^{-1}(V)$.
   (Here $f^{-1}(U) := \{x \in S \mid f(x) \in U\}$. Do not assume that an inverse function exists.)

3. If $f : S \to T$ and $g : T \to U$ are functions with inverses $a : T \to S$ and $b : U \to T$ respectively, prove that $g \circ f$ has an inverse, namely $a \circ b$.

4. Let $A$ be a set and $PA$ its power set, namely the set whose elements are subsets of $A$. Prove that for $S, T \in PA$,

   $$S \sim T \iff \text{there exists a bijection } f : S \to T$$

   defines an equivalence relation $\sim$ on $PA$.

5. Recall that addition of natural numbers is defined by the rule $m + 1 := m'$ and $m + n' := (m + n)'$. Prove that cancellation holds, that is, for all $m, n, k \in \mathbb{Z}$, $m + k = n + k$ implies $m = n$.

6. Suppose that $a, b, c$ are natural numbers so that $a \mid b$ and $b \mid c$. Prove that $a \mid c$.

7. Suppose that $p, q \in \mathbb{N}$ are prime and $p \neq q$. Show that there exist $a, b \in \mathbb{Z}$ such that $ap + bq = 1$. 