1. Let $S$ be a set and $G$ a group. Denote by $F(S,G)$ the set of all functions $f : S \rightarrow G$. Define a binary operation on $F(S,G)$ and prove that it is a group. Interpret $F(S,G)$ in the case $S = \mathbb{R}$, $G = \mathbb{R}$.

2. (a) Recall that $\Sigma_n$ denotes the group of all bijections $f : \{1, \ldots, n\} \rightarrow \{1, \ldots, n\}$. Prove that \( \{f \in \Sigma_n \mid f(1) = 1\} \) is a subgroup.
   
   (b) Let $S$ be any set, $x \in S$. Recall that the set $\text{Bij} S$ of all bijections $S \rightarrow S$ is a group (with $\circ$). Prove that \( \{f \in \text{Bij} S \mid f(x) = x\} \) is a subgroup.

3. It was proved in class that all subgroups of $\mathbb{Z}$ (with $+$) are of the form $k\mathbb{Z}$, where $k \in \mathbb{Z}$. State and prove a similar description of all subgroups of $\mathbb{Z} \times \mathbb{Z}_2$. Hint: draw some pictures first.

4. If $G$ is a finite group and $\#G$ is even, prove that $G$ contains an element besides the identity which equals its own inverse. (Hint: consider $\{g \in G \mid g \neq g^{-1}\}$; how many elements does it have?)

Now read §1.1 of Dummit & Foote and do exercises 18, 25, and 29 there.