1. List all 12 elements of $A_4$. Show that $A_4$ is not simple by exhibiting a proper normal subgroup. Hint: look back at the classification of groups of order 12.

2. (a) For $n \geq 3$, show that $\Sigma_n$ has no normal subgroup of order 2. 
   (Careful, it says order 2, not index 2!)
   (b) For $n \geq 5$, show that $A_n$ and 1 are the only proper normal subgroups of $\Sigma_n$.
   Hint: use the Second Isomorphism Theorem.

3. For any $\sigma \in \Sigma_n$, show that the sign $\varepsilon(\sigma)$ equals the determinant of its permutation matrix. Hint: use facts about conjugacy and transpositions to reduce to the case of (12).

4. (a) Show that the center of a direct product is the direct product of the centers:
   $$Z(G_1 \times \cdots \times G_n) = ZG_1 \times \cdots \times ZG_n.$$ 
   (b) Show that $G_1 \times \cdots \times G_n$ is abelian if and only if each $G_i$ is.

5. (a) For $A$ an abelian group and $n \in \mathbb{N}$, show that the set $A(n)$ of all elements whose order is finite and divides $n$ is a subgroup.
   (b) If $A \cong B$, show that $A(n) \cong B(n)$.
   (c) Show that $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_4 \not\cong \mathbb{Z}_4 \times \mathbb{Z}_4$.

6. Show, using the classification of finite abelian groups, that there are exactly 10 isomorphism classes of abelian groups of order 400. Hint: what are the possible Sylow subgroups?

7. Show, using the classification of finite abelian groups, that any finite abelian group is either cyclic or contains a subgroup isomorphic to $\mathbb{Z}_p \times \mathbb{Z}_p$ for some prime $p$. 