1. Prove that the center $ZG$ is a normal subgroup of $G$.

2. Let $T$ be a regular tetrahedron. Following the example of the cube done in class, (a) using the counting formula, determine the order of the group $G$ of rotations of $T$, (b) using Burnside’s lemma, compute the number of ways to color the faces of $T$ with 3 colors, up to rotational equivalence; (c) same thing for $n$ colors.

3. Let $m \leq n \in \mathbb{N}$. Say that a subset $S \subset \{1, \ldots, n\}$ is fixed by a permutation $\sigma \in \Sigma_n$ if $\sigma(S) = S$. Prove that the average, over all $\sigma \in \Sigma_n$, of the number of subsets with exactly $m$ elements fixed by $\sigma$ is 1. (Of course, for many $\sigma$ the number will be 0.)

4. (a) Generalizing problem 1(b) from last week, prove that the number of conjugacy classes in the dihedral group $D_{2n}$ is $(n+3)/2$ if $n$ is odd, $(n+6)/2$ if $n$ is even. What are they?

(b) Prove that the set $\{(g, h) \in D_{2n} \times D_{2n} \mid gh = hg\}$ of commuting pairs in $D_{2n}$ contains exactly $n^2 + 3n$ elements if $n$ is odd, $n^2 + 6n$ elements if $n$ is even.

Also, from Dummit & Foote, do the following problems.
From §3.1 (p. 89): 36.
From §4.3 (p. 130): 5, 6, 13.