In problems 1–5, write careful proofs of the statements given. Follow the style used in class consisting of complete sentences in the English language. Every statement should have a reason, which is either a definition or a proposition stated in class. Do not use truth tables except where asked to do so. Hint: to prove $\iff$, it suffices to prove first $\implies$ and then $\impliedby$.

1. $P \lor \neg P$ (regardless of the truth value of $P$ . . .)

2. $\neg (P \land Q) \iff (\neg P \lor \neg Q)$ (regardless of the truth values of $P$ and $Q$ . . .)

3. $(P \implies Q) \iff (\neg Q \implies \neg P)$ (regardless . . .)

4. Proof by cases: if $P \implies R$, $Q \implies R$, and $P \lor Q$ are all true, then so is $R$.

5. $((P \lor Q) \lor R) \iff (P \lor (Q \lor R))$. Also make a truth table with 8 rows showing all possibilities for $P, Q, R, P \lor Q, Q \lor R$, and the two sides of the biconditional above.

6. Write the negation of the statement $\forall x \in S \exists y \in T [(P(x, y) \land Q(x, y))]$ in terms of $\neg P(x, y)$ and $\neg Q(x, y)$. Make up a sentence in natural language (i.e. everyday speech) that has this form.

Hint for problems 7–11: use the definition of set equality.

7. Suppose $S$ is any set.
   (a) Prove that $\emptyset \subset S$.
   (b) Is it necessarily true that $\emptyset \in S$? Give either a proof or a counterexample (i.e. a specific $S$ where you prove $\emptyset \notin S$).
   (c) Is it necessarily true that $\emptyset \notin S$? Again, give either a proof or a counterexample.
   (d) Prove that if $S \subset \emptyset$, then $S = \emptyset$.

8. Prove that $\{1, 2\} = \{2, 1\}$.

9. For a set $A$, prove that $A \cup \emptyset = A$.

10. For sets $A, B, C$, prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. Also illustrate this with a Venn diagram.

11. For sets $A, B$, prove that $A = (A \setminus B) \cup (A \cap B)$. Also illustrate this with a Venn diagram.