Columbia University
Algebraic Geometry Seminar

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Gröbner geometry of flag varieties

According to Borel's theorem, the cohomology ring of the variety of flags in $\mathbb{C}^n$ is a quotient of $\mathbb{Z}[x_1, \dots, x_n]$, where x_i is the Chern class of the i^{th} standard line bundle. Lascoux and Schützenberger, based on ideas of Demazure and Bernstein-Gelfand-Gelfand, discovered polynomials that map to Ehresmann's basis of Schubert classes and are defined uniquely by their stability properties. Miraculously, these "Schubert polynomials" have positive coefficients, even though the classes x_i are not themselves positive while the Schubert classes are. I will present joint work with Allen Knutson that provides a geometric explanation for the positivity of Schubert polynomials, by reducing ordinary cohomological statements about the flag variety to equivariant cohomological statements about the vector space of $n \times n$ matrices. In particular, the coefficients of Schubert polynomials count subspaces in flat deformations of $B \times B$-orbit closures ("matrix Schubert varieties") inside the $n \times n$ matrices. At the end, I might describe a current project with Jason Starr concerning how to get explicit descriptions of certain moduli spaces of rational curves in toric varieties and (partial) flag varieties by considering appropriate analogues of the transition from the flag variety to $n \times n$ matrices.

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2:30pm

Mathematics 417