Contracting divisors with $\text{NEF}$

\[ H_{\text{MF}} \cap \text{KRF} : (X, m) \] projective manifold

\[ g_{ij} = -R_{ij}, \quad \omega = \sqrt{-1} \partial \bar{\partial} \omega, \quad [\omega] = c_1(L) \quad \text{ample} \]

\[ \omega = -\text{Ric} \Rightarrow [c_1,L] = [\omega] + c_1(K_X) \]

\[ (\text{Im}(C_\omega)): \text{Flow exists as long as } [\omega] + c_1(K_X) > 0 \]

Say $T < \infty$ s.t. $[\omega] + Tc_1(K_X)$ not positive.

\[ T \in [c_1(K_X)] \quad \text{curve} \quad C(t) = [\omega(t)] = [\omega(0)] + tC_\omega \quad \text{for} \quad t \geq 0 \]

Known as: $T \in Q$, so $[\omega] + Tc_1(K_X) = [L + TK_X] \in \text{Pos}$

and semi-ample (big f.)

\[ \Rightarrow [L + TK_X] \text{ gives map } \pi : X \to \mathbb{P}^n \]

\[ (L + TK_X), C = 0 \Rightarrow \text{map contracts } C \text{ at } (\text{very}) \cdot \text{easy}. \]

(map doesn't see $C$)

\[ \text{NE}(X) \neq \emptyset \quad \text{(very nice case).} \quad \text{NE}(X) \subset X \subset (\text{divisors/quas.}) \]

Planes $[\omega(t)]$ move until hit extremal ray.

(Will happen if $K_X$ not nef: def. of extremal rays.)

In fact, do whole wall.

If starting class non-unique, hit a wall of many dimension in $\text{NE}(X) \Rightarrow$ contract exactly one ray.

Do the project $C$ for any $y$ positive closed.
Problems:

1. What’s $\pi_1$? - Connection, fibration... $\dim > 3$?

2. $\pi_1(X)$ of singular variety? What’s nice: flow? (Wich KRF)

3. - Tian: do degenerate RF on $X$ but that’s not a step... (Siy: II)


5. - Can blow down to singular but need step? (Siy)

6. Make analysis -> limiting metric descends, nice enough to compactify?

7. Make $A_1$.

8. We get $\mathbf{C}^*$ convergence (working out metric completion $(x) = y$)


10. Canonical surgery = do it yourself.

Def: Contractible Sufficient Contraction

Let $\omega = \sum \lambda_i \omega_i$ be a Kähler form, 

$\omega_{Kähler} = \sum_{i=0}^{\infty} \omega_{Kähler}^i 
= \sum_{i=0}^{\infty} (I - (\tau+\omega)^i) \omega_{Kähler}^i$

Let $v(t) = \omega + tv_0$ and $v(0) = \omega$ be RF.

(Let's just do analysis and see what we can recover of $Y$)

Let $\pi: X \to Y$ be a contraction $C \equiv \partial Y$, we have $K(X, \omega) + T(K(X, \omega)) = \left[ \pi \times C \right]$. 

and take $\omega = (I - (\tau+\omega)^i) \omega_{Kähler}^i$. 

$\omega(0) = \omega_{Kähler}$ be RF. 

$\bar{\partial} = \log(\tau + \omega_{Kähler} + i\cdot \omega, \omega_{Kähler})$. 

$\psi = \log(\tau + \omega_{Kähler} + i\cdot \omega, \omega_{Kähler})$. 

(Defining Section for Effective Divisor, etc. Part 1 & 2.

Let's just do analysis and see what we can recover of $Y$.)
To get inside a rectangle and show:

\[ \text{uniform convergence for } t \in [0, T] : (\text{Theorem } 1) \]

1. \[ \|u(t)\| \leq C \]
   \[ u \leq C \]
   \[ C \leq C \]
   \[\text{and } u \to 0 \text{ pointwise, both } T \to 0 \text{ and } t \to 0.\]

(proof: use prime, etc.)

2.1. (1) \[ w \geq C T^{-1/2} \mu \] (best one can bound evel's form below)

For \( K = X \times T \) get \( w \to c_0 \) both \& converges in \( L^1 \| K \to 0 \)

Proof: Schauder - (ii-iv)

\[ \text{well defined; condition } A \text{ is not violated.} \]

Can: \( w \) satisfies "condition A" for \( K \) (they fit into)

If:
\[ w \geq -K \leq \int \omega = \int K.t \text{ (some } K) \]

just \( b/c \) it algebraic \( \omega \) gives finite pursuit \( w \text{ (conf.)} \)

(c works for smooth pursuit)\( k \) = \( K \) (case 1)

Rank: This all works for \( t \ni -T \text{ point} \& ( \omega (t) \text{ finite}) \)

smooth, \( Q \)-fact, terminal cases (RF limit)

Paper II: smooth case holds

This is already enough to continue the flow using weakly

\[ \text{So } T. \]
First remark that since $\Delta \phi / \epsilon^2 \to 0$ \( \epsilon = 0 \),

\( \frac{1}{\epsilon^2} \Delta \phi = O(1) / \epsilon < 30 \), so no max. point.

\[ \Rightarrow \phi_T = c \ln \epsilon \]

\[ \Rightarrow \phi_T \to \phi_T \text{ blow-up} \]

\[ \text{plumpify } \ell = \ell' \text{ theory } K_0 \text{ (ill-posed)} \]

\[ \text{closed, pos. 1,1 current} \]

\( \text{needed to } \phi_T \text{ flow, for flow } \]

\[ (\frac{\partial}{\partial t}) \phi_T \in L^p(\epsilon) \]

\[ \text{as before } b/c \omega(\epsilon) < J \text{ on } x \]

\[ \left( \int_{S_{1,0}} \left( \frac{\partial}{\partial t} \phi_T \right)^p \right) \leq C \int_{S_{1,0}} \left( \frac{\partial}{\partial t} \phi_T \right)^p \left( \frac{\partial}{\partial t} \phi_T \right)^p \leq 0 \]

\[ \Rightarrow \text{flow continuous, smooth flow only for } t > T \]

\( \text{S.W. II: } \text{becomes nondegenerate, smooth ansatz field} \)

\[ \text{metric for } t > 0 \text{ (better than deg. flow or singular } x \) \]

MA eqn changes, and thus \( \phi_T \) restricted flow

\[ \text{at } t = T \text{ (initial)} \]

Need higher order, need to control derivatives & curv. as

\[ x - T^+ \text{, in order to have a chance to control the approximating} \]

\[ \text{flows on the other side as } t \to T^+, \text{ to show } \phi(\epsilon) \text{ solves} \]

\( \text{in the other style } x = T^+ \text{, RF at } t = T \text{ (produced } \phi(\epsilon) \text{ exists)} \)
Let's solve the given problem.

1. **Problem Setup**
   - Consider the function $f(x) = \frac{1}{x} + \frac{1}{2}$. We need to find its derivative $f'(x)$.
   - Using the derivative rules, we get $f'(x) = -\frac{1}{x^2} + \frac{1}{2}$.
   - Setting $f'(x) = 0$, we find $ \frac{1}{x^2} = \frac{1}{2}$.
   - Solving for $x$, we get $x^2 = 2$, so $x = \sqrt{2}$.
   - Therefore, the critical point occurs at $x = \sqrt{2}$.

2. **Second Derivative Test**
   - Calculating the second derivative, $f''(x) = \frac{2}{x^3}$.
   - Evaluating at $x = \sqrt{2}$, we find $f''(\sqrt{2}) = \frac{2}{(\sqrt{2})^3} = \frac{1}{\sqrt{2}} > 0$.
   - Hence, the point $x = \sqrt{2}$ is a local minimum.

3. **Conclusion**
   - The function $f(x) = \frac{1}{x} + \frac{1}{2}$ has a local minimum at $x = \sqrt{2}$.
Lemma 3.2: \( \exists \) uniform \( C > 0 \), \( \forall \varepsilon \in \{0, \varepsilon\} \),
\[
\int_0^1 w(t) \, dt \leq C (T-t)^{\varepsilon/3}
\]

Proof: Assume \( \varepsilon = 0^+ \), \( \dim X = 2 \).
\[
\int \omega(t) = \frac{1}{\varepsilon} \int \frac{(T-t)\omega(t) + t \pi \gamma \omega(t)}{\varepsilon} \geq \frac{T-t}{\varepsilon} \int \omega
\]

Let \( \varepsilon = (T-t)^{\varepsilon/3} \) small. (Statement was equivalent to \( \varepsilon \) small.)

Assume \( \omega \) is a smooth function supported in \( (T-t)^{\varepsilon/3} \).

By compactness, \( \omega \) is uniformly equivalent to restriction of \( \omega \) to \( B_{1/2}(0) \) after any rotation. (Putting \( \gamma = B_{1/2}(0) \)).

Moreover, assume \( \varepsilon = 0^+ \), \( \omega = (T-t, 0, t \pi \gamma) \), \( 0 < \varepsilon < 1 \),

and \( R = \{ 0 \leq x \leq x_0, -y \leq y \leq 1 \} \subset B_{1/2}(0) \).

\[
\int p \omega \, dx \leq \int_0^{x_0} \omega \, dx \leq C (T-t)^{\varepsilon/3}
\]

Let \( y \in (p_0, p) \) be a point such that
\[
\int_0^{x_0} \omega \, dx \leq C (T-t)^{\varepsilon/3}
\]

(MVT for integrals)

\[
\| \omega \|_{L^1} = \int_0^{x_0} \sqrt{g(x, \omega) (x, y)} \, dx
\]

\[
= \left( \int_0^{x_0} \sqrt{g(x, \omega) (x, y)} \, dx \right)^{1/2} \left( \int_0^{x_0} \omega (x, y) \, dx \right)^{1/2}
\]

\[
\leq C (T-t)^{\varepsilon/3}
\]
For the other two sides, have to prove inside $C$.

$$d(x, y) + d(y, z) \leq d(x, z).$$

$$d(x, y) + d(y, z) \geq 2d(z, y).$$

(1) by key assumption L.7.7.(i).

We conclude $T(1)$.

For $\mathbb{R}^n$, take a $P$ contrary. I

(1) 2021

Some have no control on set at $E$ except via its integrals

(2) could have put one disk where circles had to go in $E$?

Cylindrical could be in the $E$ distance too.

Meters up $X \times E)_{t^1} = (y, t^1).$ Can you have $\partial$? $d^2 \delta^2$ go missing

(3) Extend $u^1$ by zero at $y^1 = d^1$.

For $H^2$, across what the image area? (needs $+$


Metric completion issue.

Right now, appears that $K(E)$ is regular, not just.

$K(E)$ can $K(E)$ be used to organize base and of data? Oppose.

RF was used to impose structure but.

on topology.

Still have (Return to p. 4 discussion of higher order.

- lost part of "canonical surgical condition."

Smoothness at $t^1 = T$ (away from $E$, $\xi$).
\[ \Delta S = (\Delta B, B) + (B, \Delta B) + 15B_1^2 + 17B_2^2 \]

\[ = - (\nabla \cdot B - B \cdot \nabla) + (15B_1^2 + 17B_2^2) + \frac{B \times R \times B}{15} \]

\[ + (\nabla \cdot B - B \cdot \nabla) \]

Since you know the norm, and \( \nabla \cdot B = \mathcal{O}(\nabla h^{-1}) = \mathcal{O}(B) = 0 \),

\[ \Delta S \geq -C \Delta S - C_0 \quad \text{for } \Delta S \quad \text{or } \Delta S \quad \text{eqn.} \]

We need to cancel \( \nabla \cdot B \) with \( \frac{\partial}{\partial t} \), under \( \nabla \). \( \text{Claim: } \frac{\partial}{\partial t} B_{1,3} = \nabla \cdot (B \cdot \nabla) E - \nabla \cdot R B \)

\[ \frac{\partial}{\partial t} B_{1,3} = g \cdot \frac{\partial}{\partial t} (g \cdot h^{-1}) g \]
and $u$-sequence.

Remark: define a section $s$ of $[G]$,

\[ \exists s: j : \mathfrak{g}(\mathfrak{g}) \to G. \]

Metric $h$ on $G$ s.t.

\[ 1 \leq \frac{1}{h_Z} \left( \frac{3 \ell_2 + 1^2}{\ell_1 + 1^2} \right) \leq \rho X r Z, \]

Also define $w = \frac{2}{\ell_1} \mathcal{O}_s (r^2 Z)$, nonvanishing orbifold $\rho X$-form,

\[ \Pi^* w = \frac{2}{\ell_1} \mathcal{O}_s \left( 3 \ell_2 \right), \]

Lemma: $w = \omega - c X \delta (h)$ is $\Pi^*$ harmonic on $X$.

And this fact is trivial. Stein $\omega$ is too hard at $0$.

Use this to argue that $\Psi \in \Omega^p (\mathcal{O}_s + \frac{2}{\ell_1} \mathcal{O}_s) \text{smooth on } X$.

So by changing within $[\mathfrak{g}(\mathfrak{g})]$, complete with smoothly $\Pi^* w$, carrying on when...
Surfaces: End game studies.

Paper II does the surf contraction that does assuming Calabi–Yau analogy = End game in Fano case.

Paper I shows general type gets contracted to smooth model. (Adjunction, $\delta$ exception and $P$'s Vare (-1).)