2. (a) \( \lim_{x \to 2} [f(x) + g(x)] = \lim_{x \to 2} f(x) + \lim_{x \to 2} g(x) = 2 + 0 = 2 \)

(b) \( \lim_{x \to a} g(x) \) does not exist since its left- and right-hand limits are not equal, so the given limit does not exist.

(c) \( \lim_{x \to 0} [f(x)g(x)] = \lim_{x \to 0} f(x) \cdot \lim_{x \to 0} g(x) = 0 \cdot 1.3 = 0 \)

(d) Since \( \lim_{x \to a} g(x) = 0 \) and \( g \) is in the denominator, but \( \lim_{x \to a} f(x) = -1 \neq 0 \), the given limit does not exist.

(e) \( \lim_{x \to a} x^3 f(x) = \left[ \lim_{x \to a} x^3 \right] \left[ \lim_{x \to a} f(x) \right] = 2^3 \cdot 2 = 16 \)

(f) \( \lim_{x \to a} \sqrt{3 + f(x)} = \sqrt{3 + \lim_{x \to a} f(x)} = \sqrt{3 + 1} = 2 \)

10. (a) The left-hand side of the equation is not defined for \( x = 2 \), but the right-hand side is.

(b) Since the equation holds for all \( x \neq 2 \), it follows that both sides of the equation approach the same limit as \( x \to 2 \), just as in Example 3. Remember that in finding \( \lim_{x \to a} f(x) \), we never consider \( x = a \).

13. \( \lim_{x \to 2} \frac{x^2 - x + 6}{x - 2} \) does not exist since \( x - 2 \to 0 \) but \( x^2 - x + 6 \to 8 \) as \( x \to 2 \).

38. We have \( \lim_{x \to 1} (2x) = 2(1) = 2 \) and \( \lim_{x \to 1} (x^4 - x^2 + 2) = 1^4 - 1^2 + 2 = 2 \). Since \( 2x \leq g(x) \leq x^4 - x^2 + 2 \) for all \( x \),

\( \lim_{x \to 1} g(x) = 2 \) by the Squeeze Theorem.

1. On the left side of \( x = 2 \), we need \( |x - 2| < |\frac{10}{3} - 2| = \frac{2}{3} \). On the right side, we need \( |x - 2| < |\frac{10}{7} - 2| = \frac{3}{7} \). For both of these conditions to be satisfied at once, we need the more restrictive of the two to hold, that is, \( |x - 2| < \frac{3}{7} \). So we can choose \( \delta = \frac{3}{7} \), or any smaller positive number.

20. Given \( \varepsilon > 0 \), we need \( \delta > 0 \) such that if \( 0 < |x - 6| < \delta \), then \( \left( \frac{x}{3} + 3 \right) - \frac{9}{2} < \varepsilon \iff \left| \frac{x}{3} - \frac{9}{2} \right| < \varepsilon \iff \frac{1}{3} |x - 6| < \varepsilon \iff |x - 6| < 4\varepsilon \). So choose \( \delta = 4\varepsilon \). Then \( 0 < |x - 6| < \delta \Rightarrow |x - 6| < 4\varepsilon \Rightarrow \frac{|x - 6|}{4} < \varepsilon \Rightarrow \left( \frac{x}{4} + 3 \right) - \frac{9}{2} < \varepsilon \). By the definition of a limit, \( \lim_{x \to 6} \left( \frac{x}{4} + 3 \right) = \frac{9}{2} \).

32. Given \( \varepsilon > 0 \), we need \( \delta > 0 \) such that if \( 0 < |x - 2| < \delta \), then \( |x^2 - 8| < \varepsilon \). Now \( |x^2 - 8| = |x - 2|(x^2 + 2x + 4) \).

If \( |x - 2| < 1 \), that is, \( 1 < x < 3 \), then \( x^2 + 2x + 4 < 3^2 + 2(3) + 4 = 19 \) and so

\( |x^2 - 8| = |x - 2|(x^2 + 2x + 4) < 19|x - 2| \).

So if we take \( \delta = \min \{ 1, \frac{19}{19} \} \), then \( 0 < |x - 2| < \delta \Rightarrow |x^2 - 8| = |x - 2|(x^2 + 2x + 4) < \frac{19}{19} \cdot 19 = \varepsilon \). Thus, by the definition of a limit, \( \lim_{x \to 2} x^2 = 8 \).

4. \( g \) is continuous on \([-4, -2), (-2, 2), [2, 4), (4, 6), \) and \((6, 8)\).
37. \( f(x) = \begin{cases} 
1 + x^2 & \text{if } x \leq 0 \\
2 - x & \text{if } 0 < x \leq 2 \\
(x - 2)^3 & \text{if } x > 2 
\end{cases} \)

\( f \) is continuous on \((-\infty, 0), (0, 2), \) and \((2, \infty)\) since it is a polynomial on each of these intervals. Now \( \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} (1 + x^2) = 1 \) and \( \lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} (2 - x) = 2, \) so \( f \) is discontinuous at 0. Since \( f(0) = 1, \) \( f \) is continuous from the left at 0. Also, \( \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (2 - x) = 0, \)

\( \lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (x - 2)^2 = 0, \) and \( f(2) = 0, \) so \( f \) is continuous at 2. The only number at which \( f \) is discontinuous is 0.

42. \( f(x) = \begin{cases} 
x^2 - 4 & \text{if } x < 2 \\
x - 2 & \text{if } 2 \leq x < 3 \\
2x - a + b & \text{if } x \geq 3 
\end{cases} \)

At \( x = 2: \)

\[ \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2^-} \frac{(x + 2)(x - 2)}{x - 2} = \lim_{x \to 2^-} (x + 2) = 2 + 2 = 4 \]

\[ \lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (ax^2 - bx + 3) = 4a - 2b + 3 \]

We must have \( 4a - b + 3 = 4, \) or \( 4a - 2b = 1 \) (1).

At \( x = 3: \)

\[ \lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (ax^2 - bx + 3) = 9a - 3b + 3 \]

\[ \lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (2x - a + b) = 6 - a + b \]

We must have \( 9a - 3b + 3 = 6 - a + b, \) or \( 10a - 4b = 3 \) (2).

Now solve the system of equations by adding \(-2\) times equation (1) to equation (2).

\[ -8a + 4b = -2 \]
\[ 10a - 4b = 3 \]
\[ 2a = 1 \]

So \( a = \frac{1}{2}. \) Substituting \( \frac{1}{2} \) for \( a \) in (1) gives us \(-2b = -1, \) so \( b = \frac{1}{2} \) as well. Thus, for \( f \) to be continuous on \((-\infty, \infty), \)

\[ a = b = \frac{1}{2}. \]

45. \( f(x) = x^2 + 10 \sin x \) is continuous on the interval \([31, 32], f(31) \approx 957, \) and \( f(32) \approx 1030. \) Since \( 957 < 1000 < 1030, \)

there is a number \( c \) in \((31, 32)\) such that \( f(c) = 1000 \) by the Intermediate Value Theorem. \( \text{Note: There is also a number } c \text{ in } \) \((-32, -31)\) such that \( f(c) = 1000. \)