2. (a) $g(x) = \int_0^x f(t)\,dt$, so $g(0) = \int_0^0 f(t)\,dt = 0$.

$g(1) = \int_0^1 f(t)\,dt = \frac{1}{2} \cdot 1 \cdot 1$ [area of triangle] $= \frac{1}{2}$.

$g(2) = \int_0^2 f(t)\,dt = \int_0^1 f(t)\,dt + \int_1^2 f(t)\,dt$ [below the x-axis]

$= \frac{1}{2} - \frac{1}{2} \cdot 1 \cdot 1 = 0$.

$g(3) = g(2) + \int_2^3 f(t)\,dt = 0 - \frac{1}{2} \cdot 1 \cdot 1 = -\frac{1}{2}$.

$g(4) = g(3) + \int_3^4 f(t)\,dt = -\frac{1}{2} + \frac{1}{2} \cdot 1 \cdot 1 = 0$.

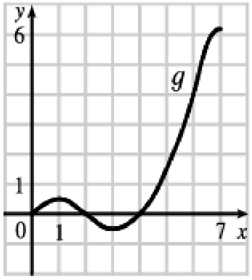
$g(5) = g(4) + \int_4^5 f(t)\,dt = 0 + 1.5 = 1.5$.

$g(6) = g(5) + \int_5^6 f(t)\,dt = 1.5 + 2.5 = 4$.

(b) $g(7) = g(6) + \int_6^7 f(t)\,dt \approx 4 + 2.2$ [estimate from the graph] $= 6.2$.

(c) The answers from part (a) and part (b) indicate that g has a minimum at $x = 3$ and a maximum at $x = 7$. This makes sense from the graph of f since we are subtracting area on $1 < x < 3$ and adding area on $3 < x < 7$.

(d)



7. $f(t) = \dfrac{1}{t^3+1}$ and $g(x) = \displaystyle\int_1^x \frac{1}{t^3+1}\,dt$, so by FTC1, $g'(x) = f(x) = \dfrac{1}{x^3+1}$. Note that the lower limit, 1, could be any real number greater than -1 and not affect this answer.

14. Let $u = x^2$. Then $\dfrac{du}{dx} = 2x$. Also, $\dfrac{dh}{dx} = \dfrac{dh}{du}\dfrac{du}{dx}$, so

$$h'(x) = \frac{d}{dx}\int_0^{x^2} \sqrt{1+r^3}\,dr = \frac{d}{du}\int_0^u \sqrt{1+r^3}\,dr \cdot \frac{du}{dx} = \sqrt{1+u^3}(2x) = 2x\sqrt{1+(x^2)^3} = 2x\sqrt{1+x^6}.$$

29. $$\int_1^9 \frac{x-1}{\sqrt{x}}\,dx = \int_1^9 \left(\frac{x}{\sqrt{x}} - \frac{1}{\sqrt{x}}\right) dx = \int_1^9 (x^{1/2} - x^{-1/2})\,dx = \left[\tfrac{2}{3}x^{3/2} - 2x^{1/2}\right]_1^9$$

$$= \left(\tfrac{2}{3} \cdot 27 - 2 \cdot 3\right) - \left(\tfrac{2}{3} - 2\right) = 12 - \left(-\tfrac{4}{3}\right) = \tfrac{40}{3}$$

41. If $f(x) = \begin{cases} \sin x & \text{if } 0 \le x < \pi/2 \\ \cos x & \text{if } \pi/2 \le x \le \pi \end{cases}$ then

$$\int_0^\pi f(x)\,dx = \int_0^{\pi/2} \sin x\,dx + \int_{\pi/2}^\pi \cos x\,dx = [-\cos x]_0^{\pi/2} + [\sin x]_{\pi/2}^\pi = -\cos\tfrac{\pi}{2} + \cos 0 + \sin\pi - \sin\tfrac{\pi}{2}$$

$$= -0 + 1 + 0 - 1 = 0$$

Note that f is integrable by Theorem 3 in Section 5.2.

12. $$\int \left(x^2 + 1 + \frac{1}{x^2+1}\right) dx = \frac{x^3}{3} + x + \tan^{-1} x + C$$

30. $\displaystyle\int_1^2 \frac{y+5y^7}{y^3}\,dy = \int_1^2 (y^{-2}+5y^4)\,dy = \left[-y^{-1}+5\cdot\tfrac{1}{5}y^5\right]_1^2 = \left[-\frac{1}{y}+y^5\right]_1^2 = \left(-\tfrac{1}{2}+32\right)-(-1+1) = \tfrac{63}{2}$

32. $\int_0^5 (2e^x+4\cos x)\,dx = \left[2e^x+4\sin x\right]_0^5 = (2e^5+4\sin 5)-(2e^0+4\sin 0) = 2e^5+4\sin 5-2 \approx 290.99$

44. $\int_0^{3\pi/2} |\sin x|\ dx = \int_0^{\pi} \sin x\,dx + \int_{\pi}^{3\pi/2}(-\sin x)\,dx = \left[-\cos x\right]_0^{\pi} + \left[\cos x\right]_{\pi}^{3\pi/2} = [1-(-1)]+[0-(-1)] = 2+1 = 3$

47. $A = \int_0^2 \left(2y-y^2\right)dy = \left[y^2-\tfrac{1}{3}y^3\right]_0^2 = \left(4-\tfrac{8}{3}\right)-0 = \tfrac{4}{3}$