Topics in Arithmetic Geometry

In this one year course in arithmetic geometry, we plane to study systematically about elliptic curves, modular curves, modular forms, and L-functions, with emphasis on BSD conjecture. Topics includes:

1. Overview of rational points on curves
2. Mordell-Weil theorem for elliptic curves
3. Hasse-Weil L-functions and the BSD conjecture
4. Complex multiplication on elliptic curves
5. Eichler Shimura theory for modular curves
6. Jacquet-Langlands theory for GL_2
7. Heights and Intersections of CM-points on modular curves
8. Gross-Zagier formula for central derivatives of L-functions.
9. Kolyvagin's system with anti-cyclotomic characters
10. Kato's system with cyclotomic characters

The material for the first six topics are standard and can be found from Silverman's books on elliptic curves and two books of Jacquet-Langlands and Jacquet on automorphic forms on GL_2. The last four topics are advanced. While their roots can be found from some papers of Gross, Zagier, Kolyvagin, Scholl, and myself, but I will try to present to class the most new progress and open questions.