An Improvement of Liouville's Theorem for Discrete Harmonic Functions

“The classical Liouville theorem says that if a harmonic function on the plane is bounded then it is a constant. At the same time for any angle on the plane, there exist non-constant harmonic functions that are bounded everywhere outside the angle.

The situation is different for discrete harmonic functions on the standard square lattices. The following strong version of the Liouville theorem holds on the two-dimensional lattice. If a discrete harmonic function is bounded on 99% of the lattice then it is constant. Simple counter-example shows that in higher dimensions such improvement is no longer true.

We will present some discrete methods, compare the behavior of continuous and discrete harmonic functions and discuss some related questions and motivation. The lectures are based on a joint work with L. Buhovsky, A. Logunov and M. Sodin.”