Ellis R. Kolchin Memorial Lecture

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"Bounding syzygies and the nature of subrings of polynomial rings"

A famous theorem of Hilbert asserts that in the polynomial ring in *N* variables over a field *K*, if one repeatedly takes modules of relations (or "syzygies"), the process terminates after at most *N* steps. One may think of having, at each step, finitely many vectors of polynomials. At the next step one replaces them with finitely many generators for the relations among these vectors. Even if one starts with relations on individual polynomials, one must deal with vectors as one iterates.

Recently, there has been a great deal of study of how many steps are needed if one starts with *n* polynomials of degree *d* but without any assumption about *N*. The question was raised by Michael Stillman, and the best answer is not known even for four quadrics or three cubics, nor is it known in general whether there is a bound that depends only on *b*, *n*.

The talk will survey what is known and discuss recent joint work with Tigran Ananyan that shows such bounds exist in degrees 2, 3, and 4. The methods used raise fundamental questions about the nature of subrings and of ideals in polynomial rings that go far beyond the original motivation.

The talk is intended for a general audience.

Thursday, May 1, 2014 at 5:30pm
Mathematics Building, Room 312
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