FALL 2019

JOSEPH FELS RITT LECTURES

Professor Neshan Wickramasekera
(Cambridge University)

Lecture 1: Variational Theory of Minimal Hypersurfaces of Riemannian Manifolds

Lecture 2: Regularity of CMC and Prescribed-mean-curvature Hypersurfaces

A fundamental idea in the study of partial differential equations is to introduce generalized solutions and establish, possibly subject to further conditions, smoothness of the generalized solutions. Having in place such a "regularity theory" is useful in many ways: besides reducing the question of existence of classical (i.e. smooth) solutions to that of existence of generalized solutions, it enables, in some instances, finding functions that are classical solutions away from a small set of non-smooth points---the best hope when everywhere smooth solutions do not exist. It also often facilitates study of weak limits of classical solutions. For much the same reasons as in PDE, it is of interest to study regularity of appropriately defined generalized submanifolds of a Riemannian manifold satisfying natural geometric constraints related to their variationally defined mean curvature. Unlike in the PDE context however, a serious difficulty in this geometric setting stems from a priori variable multiplicity of the generalized submanifolds; in fact in arbitrary codimesion, many regularity issues related to multiplicity remain poorly understood.

These lectures will describe progress made in the past several years for a large class of hypersurfaces where it is shown that this multiplicity issue has a satisfactory answer. The work culminates in a sharp regularity and compactness theory subject to certain structural conditions on the hypersurfaces and appropriate control on their mean curvature, mass and the Morse index (with respect to the relevant functional). The work includes minimal (i.e. zero mean curvature) and constant mean curvature (CMC) hypersurfaces as important special cases. We will also discuss applications of the theory including a streamlined PDE theoretic alternative to the classical Almgren--Pitts min-max existence theory for minimal hypersurfaces.

The first lecture, intended for a general mathematical audience, will focus on the minimal hypersurface theory in fairly broad terms. The second lecture will discuss key differences for general mean curvature constraints (focusing on CMC) and some aspects of the proofs of the main theorems in all cases. The lectures will in part be based on a series of speaker's works some of which are separate joint projects with C. Bellettini, O. Chodosh and Y. Tonegawa.

LECTURE 1 WEDNESDAY, OCTOBER 23, 2019
ROOM 520 FROM 4:30 - 5:30PM

LECTURE 2 THURSDAY, OCTOBER 24, 2019
ROOM 417 FROM 5:30 - 6:30PM