Ancient solutions to nonlinear diffusion equations

PDE - GR8429

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Some of the most important problems in evolution partial diferential equations are related to the understanding of *singularities*. This usually happens through a blow up procedure near the potential singularity which uses the scaling properties of the partial differential equation involved. In the case of a *parabolic* equation the blow up analysis often leads to special solutions which are defined for all time $-\infty < t \leq T$ for some $T \leq +\infty$. We refer to them as *ancient* solutions. The classification of such solutions often sheds new insight to the singularity analysis. In some flows it is also important for performing *surgery* near a singularity.

In this course we will discuss *uniqueness theorems* for ancient solutions to non-linear partial differential equations and in particular to geometric flows such as the Mean curvature flow, the Ricci flow and the Yamabe flow. This often involves the understanding of the geometric properties of such solutions. We will also discuss the *construction* of new ancient solutions from the *parabolic gluing* of solitons.

Emphasis will be given to the PDE techniques which have been developed to study these equations, as they have a wider scope of applicability beyond the special problems discussed in this course.

The following is a **brief outline** of the course which may change depending on the students demands and as the course progresses:

(1) Introduction.
(2) Ancient solutions to the semi-linear heat equation.
(3) Liouville theorems for the Navier-Stokes equations.
(4) The classification of ancient compact solutions to curve shortening flow.
(5) Ancient compact non-collapsed solutions to Mean curvature flow.
(6) The construction of ancient solutions to the Yamabe flow.
(7) The classification of ancient compact solutions to the Ricci flow on S^2.

Background: A basic courses on: (i) elliptic and parabolic PDE and (ii) Differential geometry.