How to classify quadrics:

1) If one variable (or more) does not appear, you have a **Cylinder**. Now assume all variables appear.

2) If you have \( A z = B x^2 + C y^2 + D \)
   \( A x = B y^2 + C z^2 + D \)
   \( A y = B x^2 + C z^2 + D \)
   then you have \( \begin{cases} \text{if } B \neq C \text{ have same sign, } & \text{elliptical paraboloid} \\ \text{if } B \neq C \text{ have opposite sign, } & \text{hyperbolic paraboloid} \end{cases} \)

3) If you have \( A x^2 + B y^2 + C z^2 = D \), we can assume \( D \geq 0 \) by replacing the equation by its negative if necessary.

   a) If \( D = 0 \):
      \( \begin{cases} A, B, C \text{ all positive or all negative: get a point} \\ 1 \text{ or } 2 \text{ of } A, B, C \text{ negative: get an elliptical cone} \end{cases} \)

   b) If \( D > 0 \):
      \( \begin{cases} A, B, C \text{ all negative: get nothing} \\ 2 \text{ of } A, B, C \text{ negative: get a hyperboloid of 2 sheets} \\ 1 \text{ of } A, B, C \text{ negative: get a hyperboloid of 1 sheet} \\ \text{all of } A, B, C \text{ positive: get an ellipsoid} \end{cases} \)
Table 12.1 Graphs of Quadric Surfaces

Ellipsoid

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1
\]

Elliptical cross-section in the plane \(z = c\)
The ellipse \(\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\) in the \(xy\)-plane

Elliptical Paraboloid

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}
\]
Part of the hyperbola \(\frac{x^2}{a^2} - \frac{z^2}{c^2} = 1\) in the \(xz\)-plane

Elliptical Cone

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}
\]
The line \(z = \frac{c}{2}x\) in the \(xz\)-plane

Hyperboloid of One Sheet

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1
\]
The parabola \(z = \frac{c}{2}y^2\) in the \(yz\)-plane
Part of the hyperbola \(\frac{x^2}{a^2} - \frac{z^2}{c^2} = 1\) in the \(xz\)-plane

Hyperboloid of Two Sheets

\[
\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1
\]
The hyperbola \(\frac{x^2}{a^2} - \frac{z^2}{c^2} = 1\) in the \(xz\)-plane

Hyperbolic Paraboloid

\[
\frac{y^2}{b^2} - \frac{x^2}{a^2} = \frac{z}{c} \quad c > 0
\]
The parabola \(z = \frac{c}{2}x^2\) in the \(xz\)-plane

Part of the hyperbola \(\frac{x^2}{a^2} - \frac{z^2}{c^2} = 1\) in the \(xz\)-plane

\(a\), \(b\), \(c\) are the semi-axes lengths.